DESIGN OF
POLYPHASE GENERATORS AND MOTORS

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DESIGN OF POLYPHASE GENERATORS AND MOTORS

BY

HENRY M. HOBART, M. INST. C.E.

Consulting Engineer, General Electric Company; Member American Institute Electrical Engineers; Institution Electrical Engineers; Institution Mechanical Engineers; Member Society for the Promotion of Engineering Education

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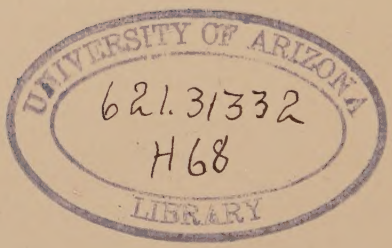

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PREFACE

During several recent years the author has given courses of lectures at three technical schools in London, on the subject of the design of electric machinery. These three schools were: the Northampton Institute of Technology; Faraday House; and University College. Various methods of procedure were employed and these ultimately developed into a general plan which (so far as it related to the subjects of Polyphase Generators and Polyphase Motors), has been followed in the present treatise. It was the author's experience that the students attending his lectures took an earnest interest in calculating designs of their own, in parallel with the working out of the typical design selected by the author for the purpose of his lectures. At the outset of the course, each member of the class was assigned the task of working out a design for a stipulated rated output, speed and pressure. Collectively, the designs undertaken by the class, constituted a series of machines, and co-operation was encouraged with a view to obtaining, at the conclusion of the course of lectures, a set of consistent designs. If a student encountered difficulty or doubt concerning some feature of his design, he was encouraged to compare notes with the students engaged in designing machines of the next larger and smaller ratings or the next higher and lower speeds. Ultimately the results for the entire group of designs were incorporated in a set of tables of which each student obtained blue prints.

At two of these three colleges, the "sandwich" system was in operation, that is to say, terms of attendance at the college were "sandwiched" with terms during which the student was employed in an electrical engineering works. The result of the author's opportunities for making comparisons is to the effect that students who were being trained in accordance with the "sandwich" system were particularly eager in working out their

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designs. Their ambition to obtain knowledge of a practical character had been whetted by their early experiences of practical work; they knew what they wanted and they were determined to take full advantage of opportunities for obtaining what they wanted. One could discuss technical subjects with them quite as one would discuss them with brother engineers. There was no need to disguise difficulties in sugar-coated pills.

It will be readily appreciated that under these circumstances there was no necessity to devote time to preliminary dissertations on fundamental principles. The author did not give these lectures in the capacity of one who is primarily a teacher but he gave them from the standpoint of an outside practitioner lecturing on subjects with which he had had occasion to be especially familiar and in which he took a deep interest. It is the function of the professional teacher to supply the student with essential preparatory information. In the author's opinion, however, a considerable knowledge of fundamental principles can advantageously be reviewed in ways which he has employed in the present treatise, namely; as occasion arises in the course of working out practical examples. Attention should be drawn to the fact that various important fundamental principles are there stated and expounded, though without the slightest regard for conventional methods.

Of the very large number of college graduates who, from time to time, have worked under the author's direction in connection with the design of dynamo-electric machinery, instances have been rare where the graduate has possessed any considerable amount of useful knowledge of the subject. Such knowledge as he has possessed on leaving college, has been of a theoretical character. Furthermore, it has been exceedingly vague and poorly assimilated, and it has not been of the slightest use in practical designing work. One is forced to the conclusion that teachers are lecturing completely over the heads of their students far more often than is generally realized. The author is of the opinion that the most effective way to teach the design of electric machinery is to lead the student, step by step, through the actual calculations, simultaneously requiring him to work out designs by himself and insisting that he shall go over each design again and again, only arriving at the final design as the result of a study of many alternatives. It is only by making shoes that one learns

to be a shoemaker and it is only by designing that one learns to be a designer.

While the author has not hesitated to incorporate in his treatise, aspects of the subject which are of an advanced character and of considerable difficulty, he wishes to disclaim explicitly any profession to comprehensively covering the entire ground. The professional designer of polyphase generators gives exhaustive consideration to many difficult matters other than those discussed in the present treatise. He must, for example, make careful provision for the enormous mechanical stresses occurring in the end connections on the occasions of short circuits on the system supplied from the generators, and when a generator is thrown on the circuit with insufficient attention to synchronizing. He will often have to take into account, in laying out the design, that the proportions shall be such as to ensure satisfactory operation in parallel with other generators already in service, and it will, furthermore, be necessary to modify the design to suit it to the characteristics of the prime mover from which it will be driven. This will involve complicated questions relating to permissible angular variations from uniform speed of rotation. The designer of high-pressure polyphase generators will find it necessary to acquire a thorough knowledge of the properties of insulating materials; of the laws of the flow of heat through them; of their gradual deterioration under the influence of prolonged subjection to high temperatures and to corona influences. He must also study the effects on insulating materials of minute traces of acids and of the presence of moisture; and, in general, the ageing of insulation from all manner of causes. The designer of polyphase generators can employ to excellent advantage a thorough knowledge of the design of fans; in fact, a large part of his attention will require to be given to calculations relating to the flow of air through passages of various kinds and under various conditions. The prevention of noise in the operation of machines which are cooled by air is a matter requiring much study. There occur in all generators losses of a more or less obscure nature, appropriately termed "stray" losses, and a wide experience in design is essential to minimizing such losses and thereby obtaining minimum temperature rise and maximum efficiency. Furthermore, mention should be made of the importance of providing a design for appropriate wave shape and insur-

ing the absence of objectionable harmonics. To deal comprehensively with these and other related matters a very extensive treatise would be necessary.

While it will now be evident that the designer must extend his studies beyond the limits of the present treatise in acquainting himself with the important subjects above mentioned, the familiarity with the subject of the design of polyphase generators and motors which can be acquired by a study of the present treatise, will nevertheless be of a decidedly advanced character. It can be best amplified to the necessary extent when the problems arise in the course of the designer's professional work, by consulting papers and discussions published in the Proceedings of Electrical Engineering Societies.

Just as the design of machinery for continuous electricity crystallizes around the design of the commutator as a nucleus, so in the design of polyphase generators, a discussion of the predetermination of the field excitation under various conditions of load as regards amount and phase, serves as a basis for acquiring familiarity with the properties of machines of this class. The author has taken the opportunity of presenting a method of dealing with the subject of the predetermination of the required excitation for specified loads, which in his opinion conforms more closely with the actual occurrences than is the case with any other method with which he is acquainted. In dealing with the design of polyphase induction motors, the calculations crystallize out around the circle ratio, and this may be considered the nucleus for the design. The insight as regards the actual occurrences in an induction motor which may be acquired by accustoming one's self to construct mentally its circle diagram, should, in the author's opinion, justify a much wider use of the circle diagram than is at present the case in America.

The author desires to acknowledge the courtesy of the Editor of the General Electric Review for permission to employ in Chapter VI, certain portions of articles which the author first published in the columns of that journal, and to Mr. P. R. Fortin for assistance in the preparation of the illustrations.

HENRY M. HOBART, M.Inst.C.E.

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DESIGN OF POLYPHASE GENERATORS AND MOTORS

CHAPTER I

INTRODUCTORY

Polyphase generators may be of either the synchronous or the induction type. Whereas tens of thousands of synchronous generators are in operation, only very few generators of the induction type have ever been built. In the case of polyphase motors, however, while hundreds of thousands of the induction type are in operation, the synchronous type is still comparatively seldom employed. Thus it is in accord with the relative importance of the respective types that the greater part of this treatise is devoted to setting forth methods of designing **synchronous** generators and **induction** motors. Brief chapters are, however, devoted to the other two types, namely **induction** generators and **synchronous** motors. The design of an induction **generator** involves considerations closely similar to those relating to the design of an induction **motor.** In fact, a machine designed for operation as an induction **motor** will usually give an excellent performance when employed as an induction **generator.** Similarly, while certain modifications are required to obtain the best results, a machine designed for operation as a synchronous **generator** will usually be suitable for operation as a synchronous **motor.**

In dealing with the two chief types; the author has adopted the plan of taking up immediately the calculations for a simple design with a given rating as regards output, speed, periodicity and pressure. In the course of these calculations, a reasonable amount of familiarity will be acquired with the leading principles involved, and the reader will be prepared profitably to consult

advanced treatises dealing with the many refinements essential to success in designing. A sound knowledge of these refinements can only be acquired in the course of the practice of designing as a profession. The subject of the design of dynamo electric machinery is being continually discussed from various viewpoints in the papers contributed to the Proceedings of engineering societies. Attempts have been made to correlate in treatises the entire accumulation of present knowledge relating to the design of dynamo-electric machinery, but such treatises necessarily extend into several volumes and even then valuable fundamental outlines of the subject are apt to be obscured by the mass of details. No such attempt is made in the present instance; the discussion is restricted to the fundamental outlines.

There are given in Appendices 1 and 2, bibliographies of papers contributed to the Journal of the Institution of Electrical Engineers and to the Transactions of the American Institute of Electrical Engineers. These papers deal with many important matters of which advanced designers must have a thorough knowledge. After acquiring proficiency in carrying through the fundamental calculations with which the present treatise deals, a study of these papers will be profitable. Indeed such extended study is essential to those engineers who propose to adopt designing as a profession.

CHAPTER II

CALCULATIONS FOR A 2500-KVA. THREE-PHASE SALIENT POLE GENERATOR

Let us at once proceed with the calculation of a design for a three-phase generator. Let its rated output be 2500 kilo olt amperes at a power factor of 0.90. Let its speed be 375 revolutions per minute and let it be required to provide 25-cycle electricity. Let it be further required that the generator shall provide a terminal pressure of 12 000 volts. We shall equip the machine with a Y-connected stator winding. The phase pressure will be:

$$\frac{12\,000}{\sqrt{3}} = \frac{12\,000}{1.732} = 6950 \text{ volts.}$$

The Number of Poles. Since the machine is driven at a speed of $\left(\frac{375}{60} = \right) 6.25$ revolutions per second, and since the required periodicity is 25 cycles per second, it follows that we must arrange for:

$$\left(\frac{25}{6.25} = \right) 4.0 \text{ cycles per revolution.}$$

Consequently we must provide four **pairs** of poles, or $(2 \times 4 =) 8$ poles.

Denoting by P the number of poles, by R the speed in revolutions per minute, and by $\sim$ the periodicity in cycles per second, we have the formula:

$$P = \frac{120 \times \sim}{R}.$$

In this treatise the power factor will be denoted by G. Since for our machine the rated load is 2500 kva. for $G = 0.90$, we may

also say that the design is for a rated output of $(0.90 \times 2500 =) 2250$ *kw.* at a power factor of 0.90.

So many alternators have been built and analyzed that the design of a machine for any particular rating is no longer a matter which should be undertaken without any reference to accumulated experience. From experience with many machines, designers have arrived at data from which they can obtain in advance some rough idea of the proportions which will be most appropriate. It is not to be concluded that the designing of a machine by reference to these data is a matter of mere routine copying. On the contrary, even by making use of all the data available, there is ample opportunity for the exercise of judgment and originality in arriving at the particular design required.

The Air-gap Diameter. Let us denote by D the internal diameter of the stator. Usually we shall express D in centimeters (cm.), but occasionally it will be more convenient to express it in millimeters (mm.), and also occasionally in decimeters (dm.), and in meters (m).

Since the internal diameter of the stator is but slightly in excess of the external diameter of the rotor, it is often convenient briefly to describe D as the " air-gap diameter," but we must not forget that strictly speaking, it is the internal diameter of the stator and is consequently a little greater than the external diameter of the rotor.

The Polar Pitch. Let us further denote the polar pitch, (in cm.), by τ. By the polar pitch is meant the distance, measured at the inner circumference of the stator, from the center of one pole to the center of the next adjacent pole. The polar pitch τ is the very first dimension for which we wish to derive a rough preliminary value. The values of τ given in Table 1 are indicated by experience to be good preliminary values for designs of the numbers of poles, the periodicity and the output shown against them in the Table.

The table indicates that the polar pitch τ should have a value of 70 cm.

$$\tau = 70.$$

Since the machine has 8 poles, the internal periphery of the stator is:

$$8 \times 70 = 560 \text{ cm.}$$

TABLE 1.—VALUES OF τ AND ξ.

	Rated Output in Kva.	$P=4$. $R=$ 750 for 25 ~ / 1500 for 50 ~		$P=6$. $R=$ 500 for 25 ~ / 1000 for 50 ~		$P=8$. $R=$ 375 for 25 ~ / 750 for 50 ~		$P=12$. $R=$ 250 for 25 ~ / 500 for 50 ~	
		τ	ξ	τ	ξ	τ	ξ	τ	ξ
25 cycles	250	56	1.12	48	1.15	44	1.20	38	1.22
	500	68	1.25	58	1.30	50	1.35	43	1.40
	1000	78	1.38	65	1.45	57	1.55	50	1.65
	2000	92	1.45	77	1.55	67	1.75	60	1.90
	4000	117	1.55	97	1.70	90	1.85	82	2.00
	6000	141	1.45	126	1.65	115	1.70	106	2.00
	8000	163	1.35	148	1.55	135	1.60	130	1.85
50 cycles	250	46	1.04	40	1.15	36	1.20	32	1.10
	500	53	1.11	46	1.30	41	1.35	38	1.25
	1000	64	1.18	55	1.45	49	1.45	49	1.50
	2000	75	1.26	65	1.50	57	1.60	51	1.70
	4000	82	1.26	75	1.60	68	1.70	60	1.78
	6000	82	1.17	82	1.55	73	1.60	66	1.85

	Rated Output in Kva.	$P=16$. $R=$ 187.5 for 25 ~ / 375 for 50 ~		$P=24$. $R=$ 125 for 25 ~ / 250 for 50 ~		$P=32$. $R=$ 93.5 for 25 ~ / 187.5 for 50 ~		$P=48$. $R=$ 125 for 50 ~		$P=64$. $R=$ 93.5 for 50 ~	
		τ	ξ	τ	ξ	τ	ξ	τ	ξ	τ	ξ
25 cycles	250	34	1.24	31	1.28	30	1.35				
	500	39	1.44	35	1.50	32	1.55				
	1000	45	1.70	40	1.75	37	1.80				
	2000	56	2.00	50	2.10	44	2.20				
	4000	76	2.10	70	2.30	62	2.40				
	6000	100	2.15								
50 cycles	250	30	1.18	26	1.20	25	1.25	25	1.30	24	1.35
	500	34	1.30	31	1.35	28	1.40	27	1.45	26	1.50
	1000	40	1.45	36	1.50	32	1.56	30	1.65	28	1.75
	2000	46	1.65	41	1.70	37	1.80	34	1.90	31	2.05
	4000	53	1.85	48	2.00	44	2.10	39	2.30	35	2.45
	6000	60	1.90	54	2.00	49	2.15				

Consequently for D, the diameter at the air gap, we have:

$$D = \frac{560}{\pi} = 178 \text{ cm.}$$

In general, we have the relation

$$D = \frac{P \times \tau}{\pi}.$$

The Output Coefficient. It will be seen that in Table 1, each value of τ is accompanied by a value designated as ξ. This quantity is termed the "output coefficient" and its general nature may be explained by reference to the following formula:

$$\xi = \frac{\text{Output in volt amperes}}{(\text{Dia. in decimeter})^2 \times (\text{Gross core length in dm.}) \times R}.$$

In Table 1, the appropriate value for ξ is given as 1.78.

$$\xi = 1.78.$$

Transposing the output-coefficient formula and substituting all the known quantities, we have:

$$\text{Gross core length (in dm.)} = \frac{2\,500\,000}{17.8^2 \times 375 \times 1.78}$$

$$= 11.8 \text{ dm.}$$

$$= 118 \text{ cm.}$$

The term λg is employed to denote the gross core length *in cm.* Thus we have:

$$\lambda g = 118.$$

D and λg are two of the most characteristic dimensions of the design, and indicate respectively the air-gap diameter and the gross core length of the stator core.

It is not to be concluded that we shall necessarily, in the completed design, adhere to the precise values originally assigned to these dimensions. On the contrary, these preliminary values simply constitute starting points from which to proceed until the design is sufficiently advanced to ascertain whether modified dimensions would be more suitable.

Discussion of the Significance of the Output Coefficient Formula. Let us consider the constitution of the formula above

given for ξ the output coefficient. Two of the terms of the denominator are D^2 and λg. Their product, or $D^2\lambda g$, is of the nature of a volume. If we were to multiply it by $\frac{\pi}{4}$ we should have the volume of a cylinder with a diameter D and a length λg. But leaving out the constant $\frac{\pi}{4}$ does not alter the *nature* of the expression $D^2\lambda g$. It is still of the nature of a volume. Thus:

$$\xi = \frac{\text{Output in volt amperes}}{(\text{A volumetric quantity}) \times R}.$$

Therefore, ξ = output per unit of a volumetric quantity ($D^2\lambda g$) and per revolution per minute (R). In other words, ξ is the output per unit of $D^2\lambda g$ per revolution per minute.

Obviously the greater the value of ξ, the more are we obtaining from every unit of volume. It is now easy to understand that in the interests of obtaining as small and low-priced a machine as is consistent with good quality, we must strive to employ the highest practicable value of ξ, the output coefficient.

While in most of our calculations we shall express D and λg in centimeters, it will occasionally be more convenient, on account of the magnitude of the results involved, to express these quantities in decimeters (as in the output coefficient formula), and, in other instances, in meters. In Table 2, are given some rough values connecting the total net weight with $D^2\lambda g$.

TABLE 2.—RELATION BETWEEN $D^2\lambda g$ AND TOTAL NET WEIGHT.

$D^2\lambda g$ (D and λg in meters).	Total Net Weight in Tons.	
	Low Speed.	High Speed.
1	26	30
2	40	50
3	50	65
5	68	85
7	80	105
10	90	125
15	108	150
20	120	
25	130	
30	135	
40	143	

The Peripheral Loading. Having determined upon the periphery ($\pi \times D$ or $P \times \tau$), the next point to be decided relates to the so-called "peripheral loading." The peripheral loading may be defined as the ampere-conductors per centimeter of air-gap periphery at the rated load of the machine. The range of appropriate values has been arrived at by experience with successive designs. Such values are given in Table 3. They are only rough indications, but they are of assistance in the preliminary stages of the preparation of a design.

TABLE 3.—THE PERIPHERAL LOADING.

Rated Output (in kva.).	Peripheral Loading, in Ampere Conductors per cm. of Periphery at the Air Gap.								
	25 Cycles.				50 Cycles.				
	$P=4$.	$P=8$.	$P=16$.	$P=32$.	$P=4$.	$P=8$.	$P=16$.	$P=32$.	$P=64$.
500	160	200	220	240	140	160	180	200	210
1 000	210	240	260	270	170	200	210	220	230
5 000	230	260	290	300	210	220	230	240	250
10 000	240	280	310	320	220	240	250	260	270

For our 8-pole, 25-cycle, 2500-kva machine we see that the values should be of the order of: 250 ampere-conductors per cm. of periphery.

Since the polar pitch, τ, is 70 cm. we should provide about: $250 \times 70 = 17\,500$ ampere-conductors per pole. Our machine is a three phaser. Consequently:

$$\text{Ampere-conductors per pole per phase} = \frac{17\,500}{3} = 5830.$$

The Current at the Rated Load. The ampere-conductors are the product of the current at rated load, I, and the number of conductors. For our 2500-kva., 12 000-volt machine, we have:

$$\text{Phase pressure} = \frac{12\,000}{\sqrt{3}} = 6950 \text{ volts};$$

$$\text{Output per phase} = \frac{2\,500\,000}{3} = 833\,000 \text{ volt-amperes};$$

$$I = \frac{833\,000}{6950} = 120 \text{ amperes}.$$

We can now ascertain the number of conductors per pole per phase by dividing the ampere-conductors by the amperes, as follows:

$$\text{Conductors} = \frac{5830}{120} = 48.6.$$

This result must be rounded off to some value which will be divisible by the number of slots per pole per phase.

The Number of Slots. The performance of a machine is in most respects more satisfactory the greater the number of slots per pole per phase. But considerations of insulation impose limitations. For 12 000-volt generators we may obtain from Table 4, reasonable preliminary assumptions for the number of slots per pole per phase, for three-phase generators, as a function of τ, the polar pitch:

TABLE 4.—RELATION BETWEEN τ AND THE NUMBER OF SLOTS PER POLE PER PHASE.

τ the Polar Pitch in cm.	Number of Slots per Pole per Phase.
20	2
25	3
30	3
35	3
40	4
50	4
60	5
80	5
100	6
120	7

Let us take for our design, 5 slots per pole per phase. Since we want some 48.6 conductors per pole per phase, let us take 10 conductors per slot and round off the number of conductors per pole per phase, to:

$$5 \times 10 = 50.$$

The Current Density. Our alternator is for 12 000 volts. At this pressure the thickness of the insulation required is so

considerable as to be a serious impediment to the escape of heat from the armature conductors. Consequently it is necessary to proportion these conductors for a lower current density than would be necessary for a machine for lower pressure. A current density of some 3.0 amperes per sq.mm. will be suitable. The current per conductor is 120 amperes. Consequently each conductor must have a cross-section of some:

$$\frac{120}{3.0} = 40 \text{ sq.mm.}$$

Let us make each conductor 12 mm. wide by 3.3 mm. high and let us arrange above each other the ten conductors which occupy each slot. Each conductor will be insulated by impregnated braid to a depth of 0.3 mm. Consequently the dimensions over the braid will be:

$$12.6 \times 3.9.$$

The ten conductors will thus occupy a space 12.6 mm. wide by

$$10 \times 3.9 = 39.0 \text{ mm. high.}$$

The Slot Insulation. Outside of this group of conductors comes the slot lining. In Table 5 are given suitable values for the thickness of the slot lining for machines for various pressures:

TABLE 5.—THICKNESS OF SLOT INSULATION.

Normal Pressure in Volts.	Thickness of Slot Lining in mm.
500	0.9
1 000	1.4
2 000	2.3
3 000	2.9
4 000	3.3
6 000	4.0
8 000	4.7
10 000	5.2
12 000	5.6

For our alternator the slot insulation should have a thickness of 5.6 mm. all round the group of conductors. The dimensions to the outside of the insulation will thus be:

$$2\times5.6+12.6=11.2+12.6=23.8 \text{ mm.},$$

by:

$$2\times5.6+39.0=11.2+39.0=50.2 \text{ mm.}$$

Thus the insulated dimensions are:

$$23.8\times50.2.$$

The Dimensions of the Stator Slot. Allowing 7 mm. at the top of the slot for a retaining wedge and allowing 0.3 mm. additional width for tolerence in assembling the slotted punchings, we arrive at the following dimensions for the stator slot:

$$\text{Depth}=50.2+7 \quad =57.2 \text{ mm.};$$

$$\text{Width}=23.8+0.3=24.1 \text{ mm.}$$

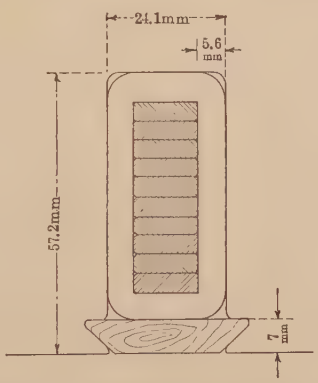


FIG. 1.—Stator Slot for 12 000-volt, 2500-kva., 375 r.p.m., 25-cycle, Three-phase Generator.

The above design for the slot is shown in Fig. 1. This can only be regarded as a preliminary design. At a later stage of the calculations, various considerations, such as tooth density, may require that the design be modified.

The Slot Space Factor. The gross area of the slot is:

$$57.2\times24.1=1380 \text{ sq.mm.}$$

Thus the copper only occupies:

$$\frac{10\times40}{1380}\times100=29.0 \text{ per cent.}$$

of the total available space.

We express this by stating that the "slot space factor" is equal to 0.29. The slot space factor is lower the higher the working pressure for which the alternator is designed.

The Magnetic Flux

Having now made provision for the armature windings, we must proceed to provide for the magnetic flux with which these windings are linked. Let us start out from fundamental considerations.

Dynamic Induction. A conductor of 1 cm. length is moved in a direction normal to its length and normal to the direction of a magnetic field, of which the density is 1 line per square centimeter. Let the velocity with which this conductor is moved, be 1 cm. per second. Then the pressure set up between the two ends of this conductor will be $1\times1\times1\times10^{-8}$ volt. If the conductor, instead of being of 1 cm. length, has a length of 10 cm. then the pressure between the ends becomes

$$10\times1\times1\times10^{-8} \text{ volts.}$$

If the field density instead of being 1 line per sq.cm., is 10 000 lines per sq.cm., then the pressure will be

$$10\times10\,000\times1\times10^{-8} \text{ volts.}$$

If the speed of the conductor is 1000 cm. per second instead of 1 cm. per second, then the pressure will be

$$10\times10\,000\times1000\times10^{-8} \text{ volts.}$$

Obviously with our 1 cm. conductor with a velocity of 1 cm. per sec., and moving in a field of a density of 1 line per sq. cm., the total cutting of lines was at the rate of:

$$1\times1\times1=1 \text{ line per second.}$$

But with our 10 cm. conductor moving in a field of a density of 10 000 lines per sq.cm., and with a velocity of 1000 cm. per sec., the total cutting is at the rate of:

$$10\times10\,000\times1000=100\,000\,000 \text{ lines per second.}$$

In other words, the pressure, in volts, at the terminals of the conductor is equal to the total number of lines cut per second, multiplied by 10^{-8}. Let us now, instead of considering a straight conductor, take a case where the conductor is bent to constitute the sides of a turn. Let this turn occupy a plane normal to a uniform flux. If in this position, the portion of the flux passing through this turn is 1 000 000 lines (or 1 megaline), then if in 1 second the coil is turned so that its plane is in line with the direction of the flux, there will in this latter position be no flux linked with the turn. The flux linked with the turn will, in 1 second, have decreased from 1 megaline to 0. The conductor constituting the sides of this turn will have cut 1 000 000 lines. If, during this 1 second, the rate of cutting is uniform, then the pressure at the terminals will be

$$1\,000\,000 \times 10^{-8} = 0.0100 \text{ volt.}$$

Thus 1 turn revolved in 1 second through 90 degrees from a position where it is linked with 1 megaline into a position in which it is parallel to the flux, will have induced in it a pressure of 0.0100 volt. Obviously at this speed, 4 seconds are required for this turn to make one complete revolution, i.e., to complete one cycle. The periodicity is thus only 0.25 cycle per second. If the speed is quadrupled, corresponding to one cycle per second, then the pressure will be

$$4 \times 0.01 = 0.0400 \text{ volt.}$$

The Pressure Formula. If instead of having one turn,we have a coil of T turns, and if this coil revolves at a periodicity of $\sim$ cycles per second, and if, instead of being linked with 1 megaline, the flux is M megalines, then the pressure at the terminals of the winding is:

$$V = 0.0400 \times T \times \sim \times M.$$

This is the general formula for the average e.m.f. induced in the windings of an alternator. But usually we are concerned with the mean effective value and not with the average value.

For a sine-wave curve of e.m.f. the mean effective value is 11 per cent greater than the average value. Consequently for the mean effective value of the e.m.f., the formula becomes

$$V = 1.11 \times 0.0400 \times T \times \sim \times M,$$

or

$$V = 0.0444 \times T \times \sim \times M.$$

Modification of Pressure Formula. The formula only holds when all the turns of the winding are (when at the position of maximum linkage of flux and turns), simultaneously linked with the total flux M. This is substantially the case with the windings of any one phase of a three-phase machine. A reference to Fig. 2 in which are drawn the windings of only one of the three phases, will show that the windings of one phase occupy only one-third of the polar pitch and so long as the pole arc of the field pole does not exceed two-thirds of the polar pitch, there is complete linkage of all the turns with the entire flux. But in a two-phase machine (often termed a quarter-phase machine), the windings of each phase occupy one-half of the polar pitch and in such a case as that shown in Fig. 3, in which are drawn the windings of only one of the two phases, it is seen that the innermost turns of one phase are not linked with quite the entire flux.

The Spread of the Winding. We express these facts by stating that in a three-phase winding, the spread of the winding is 33.3 per cent of the polar pitch and that in a two-phase winding the spread of the winding is 50 per cent of the polar pitch. In view of the allowances which must sometimes be made for incomplete simultaneous linkage of flux and turns, it is preferable to replace the formula:

$$V = 0.0444 \times T \times \sim \times M$$

by the formula:

$$V = K \times T \times \sim \times M.$$

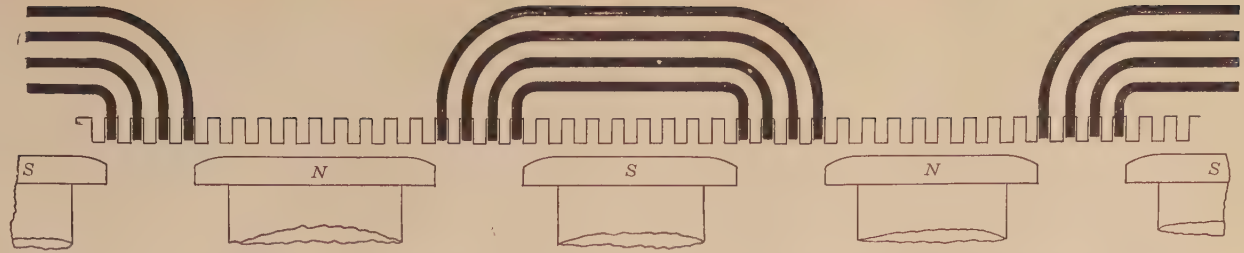


FIG. 2.—Diagrammatic Representation of the Windings of One Phase of a Three-phase Generator.

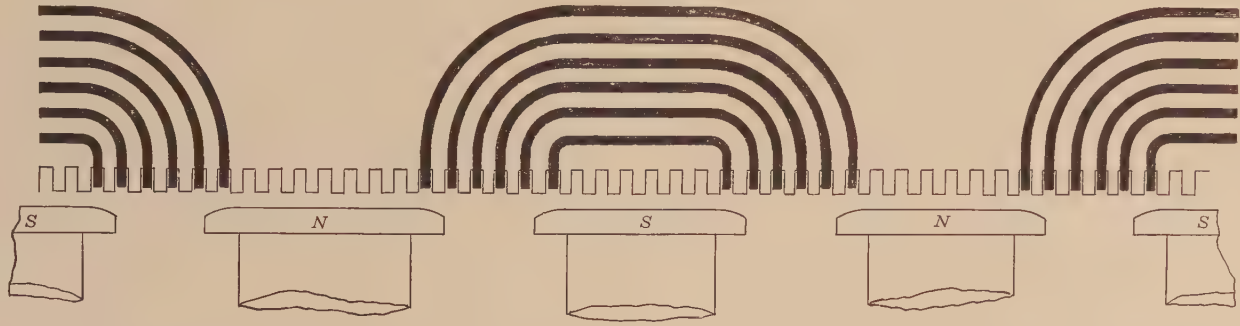


FIG. 3.—Diagrammatic Representation of the Windings of One Phase of a Two-phase (Quarter-phase) Generator.

The coefficient K for various winding spreads and for various ratios of pole arc to pitch may be obtained by reference to the curves in Fig. 4.

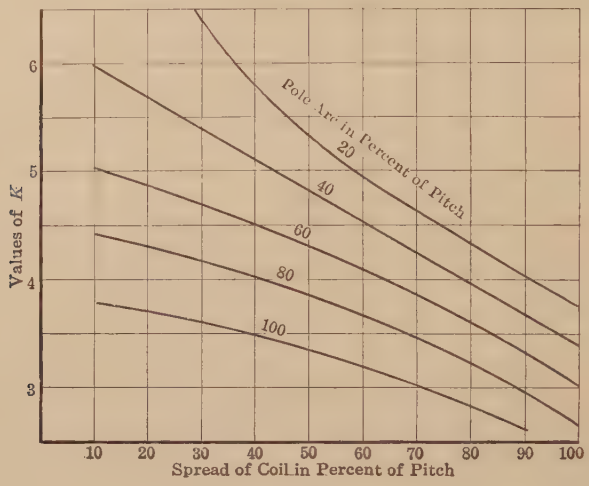


FIG. 4.—Curves for Obtaining Values of K in the Formula $V = K \times T \times \sim \times M$.

Full and Fractional Pitch Windings. In the above statements, it has been tacitly assumed that the windings are of the full-pitch type. In a full-pitch winding, the center of the left-hand side of a group of armature coils is distant by τ, the polar pitch from the center of the right-hand side of the group of armature coils. Thus in Fig. 5, is shown a full-pitch winding. But it is sometimes desirable to employ windings of lesser pitch. These are called fractional pitch windings. In Figs. 6 to 11, are shown windings in which the winding pitches range from 91.5 per cent down to 50 per cent of the polar pitch. For such windings it is necessary to introduce another factor into the pressure formula and we can employ the factor K' which we may term the winding-pitch factor. If the winding pitch is x per cent then K' will be equal to

$$\sin\left(\frac{x}{100}\times 90^\circ\right).$$

FIG. 5.—Diagrammatic Representation of a Full-pitch, Three-phase Winding.

FIG. 6.—Diagrammatic Representation of a Three-phase Winding in which the Winding Pitch is 91.5 per cent of the Polar Pitch.

FIG. 7.—Diagrammatic Representation of a Three-phase Winding in which the Winding Pitch is 83.5 per cent of the Polar Pitch.

FIG. 8.—Diagrammatic Representation of a Three-phase Winding in which the Winding Pitch is 75 per cent of the Polar Pitch.

FIG. 9.—Diagrammatic Representation of a Three-phase Winding in which the Winding Pitch is 66.7 per cent of the Polar Pitch.

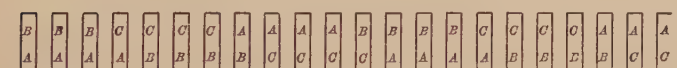


FIG. 10.—Diagrammatic Representation of a Three-phase Winding in which the Winding Pitch is 58.5 per cent of the Polar Pitch.

FIG. 11.—Diagrammatic Representation of a Three-phase Winding in which the Winding Pitch is 50.0 per cent of the Polar Pitch.

Thus when the winding pitch is 80 per cent, the winding-pitch factor is equal to

$$\sin\left(\frac{80}{100}\times 90^\circ\right) = \sin 72^\circ = 0.951.$$

Digression Regarding Types of Windings. An examination of Fig. 5 will show that the conductors contained in any one slot belong exclusively to some one of the three phases. On the contrary in the case of fractional-pitch windings, there are a certain proportion of the slots containing conductors of two different phases. As a consequence, fractional-pitch windings should be of the two-layer type. This leads to the use of "lap" windings such as are employed for continuous-electricity machines. But *full-pitch* windings may be carried out either as "lap" windings or as "spiral" windings, and consequently as single-layer or as two-layer windings.

The two varieties are shown diagrammatically in Figs. 12 and 13 on the following page.

While dealing with this subject of windings the occasion is appropriate for explaining another terminological distinction. Windings may be carried out in the manner termed "whole-coiled," in which, for any one phase, there is an armature coil opposite each field pole, as indicated in Fig. 14, or they may be carried out as half-coiled windings, there being, for any one phase, only one armature coil per **pair of poles** instead of one armature coil **per pole.** A half-coiled winding is shown in Fig. 15. It is usual to employ half-coiled windings for three-phase generators where the spiral type is adopted. But lap windings are inherently whole-coiled windings as may be seen from an inspection of Fig. 12.

It is mechanically desirable to employ fractional-pitch windings in bipolar designs, as otherwise the arrangement of the end connections presents difficulties. In most instances of designs for more than two poles, a full-pitch winding is quite as suitable as any other. Let us employ a full-pitch winding in our 2500-kva. machine.

Estimation of the Flux per Pole. We can now estimate the flux required, when, at no load, the terminal pressure is 12 000 volts. This pressure corresponds to

$$\frac{12\,000}{\sqrt{3}} = 6950 \text{ volts per phase.}$$

FIG. 12.—6-Pole Lap Winding with 72 Conductors.

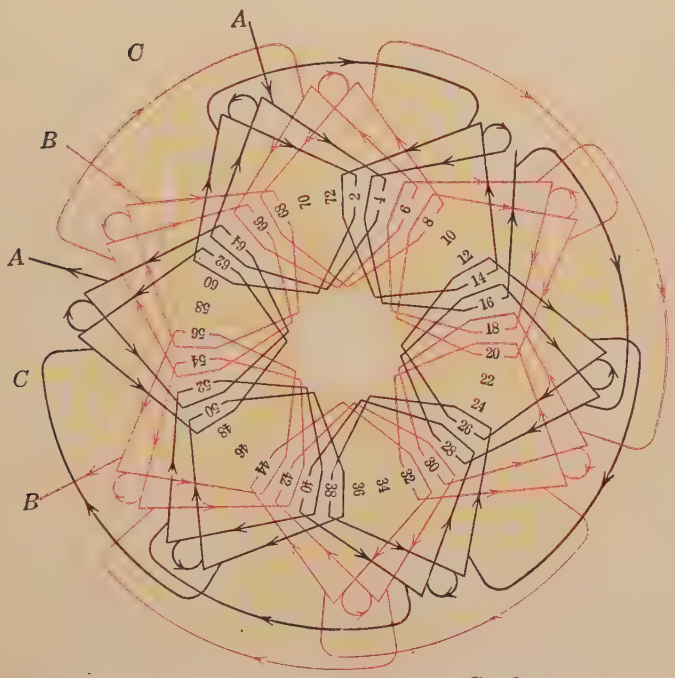


FIG. 13.—6-Pole Spiral Winding with 72 Conductors.

Therefore

$$V = 6950.$$

We have determined upon employing 10 conductors per slot and 5 slots per pole per phase. Consequently for T, the number of turns in series per phase, we have, (since the machine has 8 poles):

$$T = \frac{8 \times 10 \times 5}{2} = 200.$$

$$\sim = 25.$$

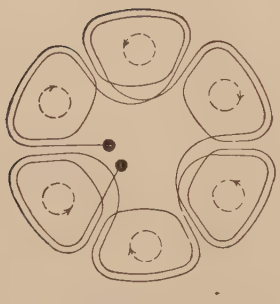


Fig. 14.—Whole-coiled Spiral Winding.

Fig. 15.—Half-coiled Spiral Winding.

Thus we have:

$$6950 = 0.0444 \times 200 \times 25 \times M.$$

Therefore

$$M = \frac{6950}{0.0444 \times 200 \times 25}$$

$$= 31.3 \text{ megalines.}$$

31.3 megalines must, at no load, enter the armature from each field pole.

The I R Drop at Full Load. At full load we must make an allowance for the IR drop in the armature winding. We have previously ascertained that the full-load current is

$$I = 120 \text{ amperes.}$$

Estimation of the Mean Length of Turn and of the Armature Resistance. We must now estimate the armature resistance. A convenient empirical formula for estimating the mean length of one turn (mlt) is:

$$\text{mlt.} = 2\lambda g + K\tau.$$

Appropriate values for K for machines for various pressures are given in Table 6:

TABLE 6.—VALUES OF K IN FORMULA FOR MLT.

Terminal Pressure for which the Stator is Wound.	Value of K in the Formula: mlt. $= 2\lambda g + K\tau$.
500	2.5
1 000	3.0
2 000	3.5
4 000	4.0
6 000	4.5
8 000	5.0
10 000	5.5
12 000	6.0

Thus for our 2500-kva. design we have:

$$\begin{aligned} \text{mlt.} &= 2\times 118 + 6.0 \times 70 \\ &= 236 + 420 \\ &= 656 \text{ cm.} \end{aligned}$$

The cross-section of the conductor is $(1.2\times 0.33 =) 0.396$ sq. cm.

It is a good plan from the standpoint of uniformity, to figure out all warm resistances (of the usual run of dynamo-electric

machinery), to correspond to a temperature of 60° Cent. At this temperature, the specific resistance of commercial copper is:

$$0.00000200 \text{ ohm per cm. cube.}$$

At 60° Cent. the resistance per phase, is, for our 2500-kva. machine:

$$\text{Resis.} = \frac{0.00000200 \times 656 \times 200}{0.396}$$

$$= 0.665 \text{ ohm.}$$

Consequently, at full load, the IR drop per phase is:

$$120 \times 0.665 = 80.0 \text{ volts.}$$

Thus at full load the internal pressure is:

$$6950 + 80 = 7030 \text{ volts per phase.}$$

At full load then, the flux entering the armature from each pole is:

$$M = \frac{7030}{0.0444 \times 200 \times 25}$$

$$= 31.6 \text{ megalines.}$$

In this particular case, the internal drop is so small that it is hardly worth while to distinguish between the full-load and no-load values of the flux per pole. But often (particularly in small, slow-speed machines), there is a more appreciable difference which must be taken into account.

The Design of the Magnetic Circuit. We are now ready to undertake the design of the magnetic circuit for our machine. In the stator, the magnetic circuit must transmit a flux of 31.6 megalines per pole. This is the flux which must become linked with the turns of the armature winding. But a somewhat greater flux must be set up in the magnet core, for on its way to the armature this flux experiences some loss through magnetic leakage. The ratio of the flux originally set up in the magnet core to that finally entering the armature is termed the leakage factor.

Leakage Factor. In our machine, we shall assume the leakage factor to be 1.15; that is to say, the quantity of flux taking its rise in the magnet core is 15 per cent greater than the portion finally entering the armature and becoming linked with the turns of the armature windings.

Consequently the cross-sections of the magnet cores and of the yoke must be proportioned for transmitting this 15 per cent greater flux.

$$1.15 \times 31.6 = 36.3 \text{ megalines.}$$

Material and Shape of Magnet Core. We shall, in this instance, make the magnet core of cast steel and we shall employ a magnetic density of 17 500 lines per sq.cm. Consequently we shall require to provide a cross-section of

$$\frac{36\,300\,000}{17\,500} = 2080 \text{ sq.cm.}$$

A circular magnet core of this cross-section, would have a diameter of

$$\sqrt{\frac{4 \times 2080}{\pi}} = 51.4 \text{ cm.}$$

At the air-gap, the polar pitch, τ, is equal to 70 cm., but it becomes smaller as we approach the inner ends of the magnet cores and there would not, in our machine, be room for cores of a diameter of 51.4 cm., even aside from the space required on these cores for the magnet windings. Let us take as a preliminary assumption for the radial length of the magnet cores (including pole shoes) 28 cm. This gives a diameter at the inner ends of the magnet cores (neglecting the radial depth of the air-gap), of

$$178 - 2 \times 28 = 122 \text{ cm.}$$

The polar pitch at this diameter is:

$$\frac{122\pi}{8} = 48.0 \text{ cm.}$$

Obviously, then, we could not employ a magnet core of circular cross-section, for we have seen that its diameter would require to be 51.4 cm. Out of the total available circumferential dimension of 48.0 cm., let us take the lateral dimension of the magnet core as 26 cm., and let us constitute the section of the magnet core, of a rectangle with a semi-circle at each end, as shown in Fig. 16. The diameter of the semi-circle is 26 cm. Consequently the area provided by the two semi-circles amounts to:

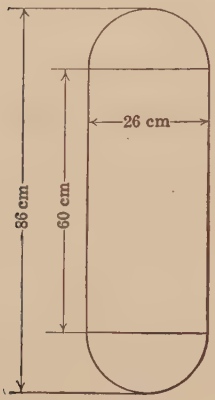


Fig. 16.—Cross-section of Magnet Core of 8-pole, 375 r.p.m., 2500-kva. Three-phase Generator.

$$26^2 \times \frac{\pi}{4} = 530 \text{ sq.cm.}$$

This leaves

$$2080 - 530 = 1550 \text{ sq.cm.}$$

to be provided by the rectangle. Thus the length of the rectangle (parallel to the shaft) must be:

$$\frac{1550}{26} = 59.6 \text{ cm. (or 60 cm.).}$$

The overall length of the magnet core is, then, $26+60=86$ cm. The pole shoe may be made 114 cm. long, thus projecting:

$$\frac{114-86}{2} = 14 \text{ cm.}$$

at each end of the magnet core. Let us make the pole arc equal to 42 cm. This is $\frac{42}{70}\left(\times 100 = \right)$ 60 per cent of the pitch at the air gap. We may allow 8 cm. for the radial depth of the pole shoe at the center. This leaves $28-8=20$ cm. for the radial length along the magnet core which is available for the magnet winding. The depth available for the winding is—at the lower end of the magnet core:

$$\frac{48-26}{2} = 11 \text{ cm.}$$

If we make our magnet spool winding of equal depth from top to bottom, then, allowing a centimeter of free space at the lower end of the spool, the two dimensions of the cross-section of the spool winding will be:

$$20 \text{ cm.} \times 10 \text{ cm.}$$

It remains to be ascertained at a later stage whether this space is sufficient to accomodate the required ampere-turns.

The Cross-section of the Magnetic Circuit at the Stator Teeth. The next part of the magnetic circuit which we should investigate, is the section at the stator teeth. We must first determine upon suitable proportions forthe ventilating ducts. A suitable number of ventilating ducts may be arrived at from the data in Table 7.

TABLE 7.—DATA REGARDING VENTILATING DUCTS.

Peripheral Speed in Meters per Second.	Number of Ventilating Ducts, Each 15 mm. Wide, per Decimeter of Gross Core Length.			
	For $\lambda g = 20$.	For $\lambda g = 50$.	For $\lambda g = 80$.	For $\lambda g = 150$.
10	2.1	2.3	2.5	2.7
20	1.5	1.8	2.1	2.3
30	1.1	1.4	1.7	1.9
40	0.8	1.1	1.4	1.6
60	0.6	0.9	1.2	1.4
80	0.4	0.7	1.0	1.2

In order to apply the data in this table, we require to know the peripheral speed.

Determination of the Peripheral Speed. Not having yet determined upon the radial depth of the air-gap, let us still neglect it and estimate the peripheral speed from the internal diameter of the stator. We have:

$$D = 178 \text{ cm.}$$

$$\text{Peripheral speed} = 1.78 \times \pi \times \frac{375}{60} = 35 \text{ meters per sec.}$$

$$\lambda g = 118.$$

Let us, then, arrange for 1.7 ventilating ducts per decimeter of gross core length.

$$11.8 \times 1.7 = 20.$$

We shall provide 20 ducts and each duct shall have a width of 15 mm. Thus the aggregate width occupied by ventilating ducts is $20 \times 1.5 = 30$ cm. But 10 per cent of the "apparent" thickness of the core laminations is occupied by layers of varnish by means of which the sheets are insulated from one another in order to prevent eddy currents.

The Net Core Length. The length of active magnetic material, after deducting the space occupied by the ventilating ducts and by the insulating varnish, may be termed the *net core length* and may be designated by λ_n. For our 2500-kva machine we have:

$$\lambda_n = (118 - 30) \times 0.90 = 79.2 \text{ cm.}$$

The width of the stator slot has been calculated and has been ascertained to be 24.1 mm. There are 15 teeth per pole (since there are 5 slots per pole per phase). The tooth pitch at the air-gap is

$$\left(\frac{700}{15} = \right) 46.6 \text{ mm.}$$

Consequently the width of each tooth is

$$(46.6 - 24.1 =) 22.5 \text{ mm.}$$

The aggregate width of the 15 teeth is

$$15 \times 2.25 = 33.8 \text{ cm.}$$

Thus the gross cross-section per pole, at the narrowest part of the stator teeth, is

$$79.2 \times 33.8 = 2680 \text{ sq.cm.}$$

But only a portion of this section will be employed at any one time for transmitting the flux per pole, for the pole arc is only 60 per cent of τ. The portion directly opposite the pole face is

$$0.60 \times 2680 = 1610 \text{ sq.cm.}$$

On the other hand, the lines will spread considerably in crossing the gap, and this spreading will increase the cross-section of the stator teeth utilized at any instant by the flux per pole. Let us in our machine take the spreading factor equal to 1.15. Consequently we have:

Cross-section of magnetic circuit at stator teeth =

$$1.15 \times 1610 = 1850 \text{ sq.cm.}$$

The density at the narrowest part of the stator teeth is:

$$\frac{31\,600\,000}{1850} = 17\,100 \text{ lines per sq.cm.}$$

This is a suitable value; indeed a slightly higher value, say anything up to 19 000 lines per sq.cm., could have been employed.

Density and Section in Stator Core. For the density in the stator core, back of the teeth, we may take 10 000 lines per sq.cm. The total flux in the stator is 31.6 megalines per pole. This flux, after passing along the teeth, divides into two equal parts and flows off to right and left to the adjacent poles on either side, as indicated diagrammatically in Fig. 17. Consequently the magnetic cross-section required in the stator core is:

$$\frac{31\,600\,000}{2\times 10\,000} = 1580 \text{ sq.cm.}$$

Since λn is equal to 79.2 cm. the radial depth of the stator core back of the slots must be

$$\frac{1580}{79.2} = 20 \text{ cm.}$$

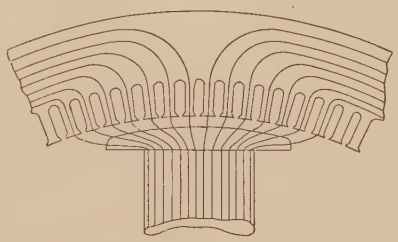


Fig. 17.—Diagrammatic Representation of the Path of the Magnetic Flux in the Magnet Core, Pole Shoe, Air-gap, Stator Teeth, and Stator Core.

The slot depth is 5.72 cm. Consequently the total radial depth of the stator core from the air-gap to the external periphery is $20.0+5.7=25.7$ cm. D, the internal diameter of the stator punching, is 178 cm. Consequently the external diameter of the stator punching is:

$$178+2\times 25.7 = 229.4 \text{ cm. (or 230 cm.)}$$

In Fig. 18 is given a drawing of the stator punching, and in Fig. 19 is shown a section through the stator core with its 20 ventilating ducts.

Even now we are not quite ready to consider the question of assigning a suitable value to the depth of the air-gap. But pending arriving at the right stage of the calculation, let us proceed on the basis that the depth of the air-gap is 2 cm. This permits us to list a number of diametrical measurements, as follows:

Preliminary Tabulation of Leading Diameters.

External diameter of stator core	2300 mm.
Diameter at bottom of stator slots (1780+2×57=)	1894 mm.
Internal diameter of stator (D)	1780 mm.
External diameter of rotor (1780−2×20)	1740 mm.
Diameter to bottom of pole shoes (1740−2×80)	1580 mm.
Diameter to bottom of magnet cores (1580−2×200)	1180 mm.

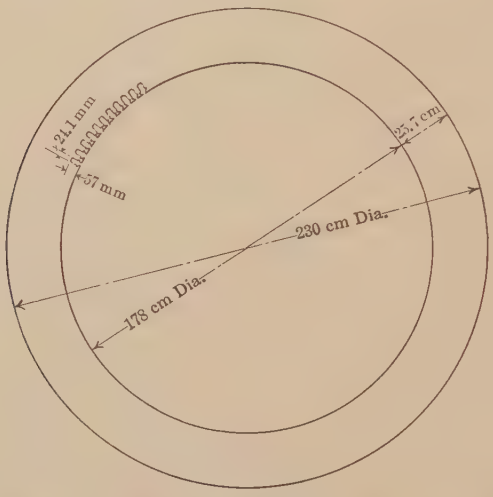


FIG. 18.—Dimensions of Stator Lamination for 8-pole, 375-r.p.m., 2500-kva., Three-phase Generator.

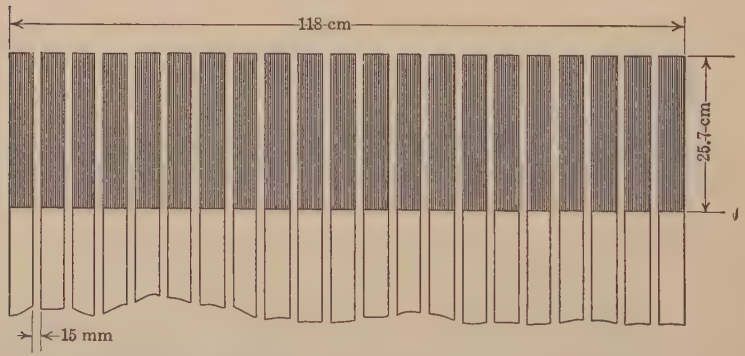


FIG. 19.—Section through Stator Core of 8-pole, 375-r.p.m., 2500-kva., Three phase Generator, showing the 20 Ventilating Ducts.

An end view of the design of the magnetic circuit so far as we have yet proceeded, is shown in Fig. 20.

We are now ready to work out the cross-section of the so-called "magnet yoke" which completes the magnetic circuit between the inner extremities of the magnet cores. We may employ a density of 12 000 lines per sq.cm. Here also the flux from any one magnet core divides into two equal halves which flow respectively to the right and to the left on their way to the

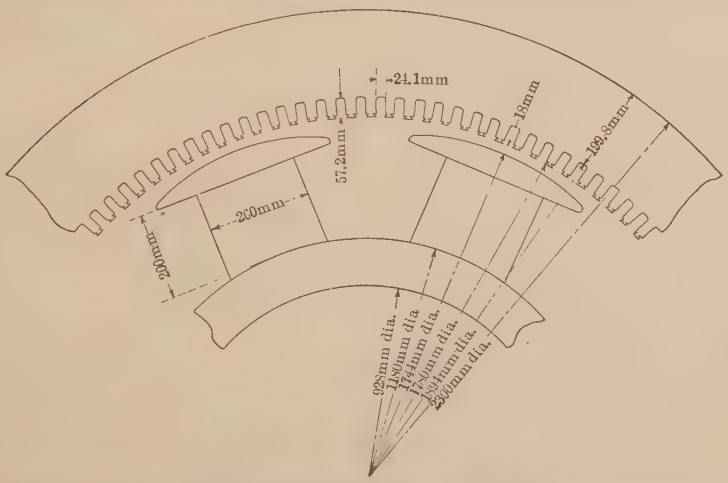


FIG. 20—End View of Design of Magnetic Circuit of 8-pole, 375-r.p.m., 2500 kva., Three-phase Alternator.

adjacent magnet cores on either side. Consequently for the magnet yoke, we require a cross-section of

$$\frac{36\,300\,000}{2\times 12\,000}=1510 \text{ sq.cm.}$$

For the dimension parallel to the shaft, let us employ 120 cm. The radial depth of the magnet yoke should consequently be:

$$\frac{1510}{120}=12.6 \text{ cm.}$$

Since the external diameter of the magnet yoke is 1180 mm., the internal diameter is $1180-2\times 126=928$ mm. The complete

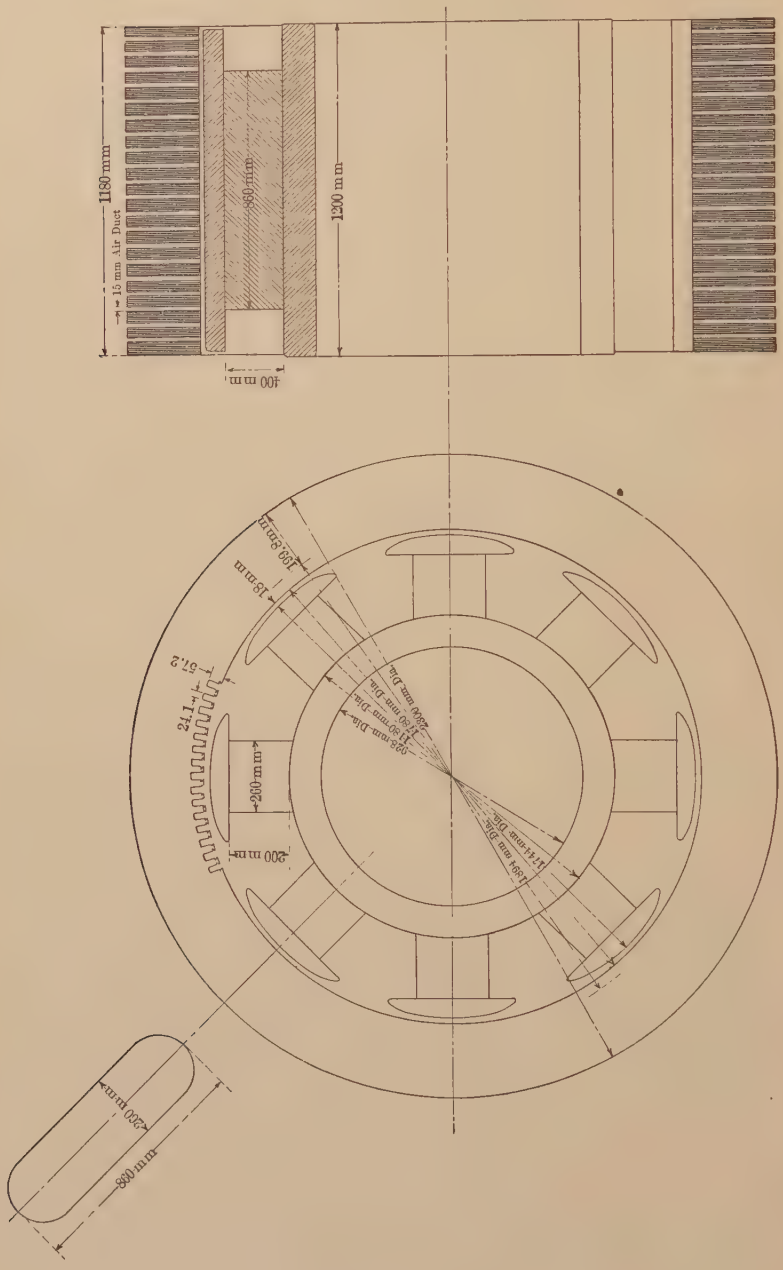


Fig. 21.—Sketches of the Disposition of the Magnetic Material in an 8-pole, 375 r.p.m., 2500-kva., Three-phase Generator.

list of leading diameters of the parts of the magnet circuit, is, (pending arriving later at a final value for the air-gap depth), as follows:

Extended Preliminary Tabulation of Leading Diameters.

External diameter of stator core	2300 mm.
Diameter at bottom of stator slots	1894 mm.
Internal diameter of stator (D)	1780 mm.
External diameter of rotor	1740 mm.
Diameter to bottom of pole shoes	1580 mm.
External diameter of magnet yoke	1180 mm.
Internal diameter of magnet yoke	928 mm.

These diameters and the leading dimensions parallel to the shaft, are indicated in the views in Fig. 21.

Mean Length of Magnetic Circuit. The next step is to ascertain the lengths of the various parts of the magnetic circuit, i.e., of the mean path followed by the magnetic lines. This mean path is indicated in Fig. 22. The mmf. in the field winding on

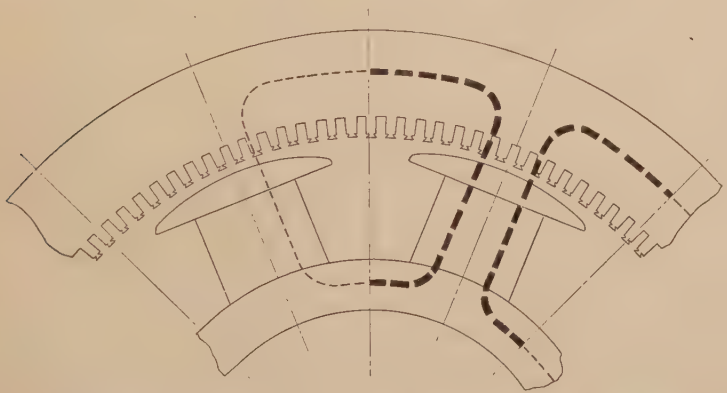


FIG. 22.—Diagram showing the Mean Path Followed by the Magnetic Lines in an 8-pole Generator with an Internal Revolving Field.

any one of the magnet poles has the task of dealing with just one-half of the complete magnetic circuit formed by two adjacent poles. The lengths which we desire to ascertain are the lengths corresponding to such a half circuit as indicated by the heavy lines in Fig. 22.

It is amply exact for our purpose to take the lengths in the stator core and in the magnet yoke as equal to the mean circumferences in those parts, divided by twice the number of poles. The mean diameters in these parts are:

$$\text{Mean diameter in stator core} = \frac{230.0+189.4}{2} = 209.7 \text{ cm.}$$

$$\text{“ “ magnet yoke} = \frac{118.0+92.8}{2} = 105.4 \text{ cm.}$$

The machine has 8 poles. Consequently:

$$\text{Mean length magnetic circuit in stator core} = \frac{209.7\times\pi}{2\times 8} = 41 \text{ cm.}$$

$$\text{Mean length magnetic circuit in magnet yoke} = \frac{105.4\times\pi}{2\times 8} = 21 \text{ cm.}$$

We also have:

Mean length of magnetic circuit in teeth = 5.7 cm.
Mean length of magnetic circuit in magnet core = 20 cm.

The pole shoe is so unimportant a part that we may in the calculation of the required magnetomotive forces (mmf.) neglect it. As to the air-gap, we must, for reasons which will be understood later, still defer taking up the calculations relating to it.

We may now make up the following Table:

Tabulation of Data for Mmf. Calculations.

Designation of Parts of Magnetic Circuit.	Lengths in Centimeters of Parts of Magnetic Circuit.	Cross-sections, in Square Centimeters of Parts of Magnetic Circuit.	Densities in Lines per Square Centimeter of Parts of Magnetic Circuit.
Teeth.....	5.7	1850	17 100
Magnet core.....	20	2080	17 500
Magnet yoke....	21	1510	12 000
Stator core.....	41	1580	10 000

Saturation Data of Magnetic Materials. In order to ascertain the mmf. required in each of these portions of the magnetic

circuit, we must consult saturation data of the materials of which these parts are built. Appropriate values are given in Table 8:

TABLE 8. MAGNETOMOTIVE FORCE PER CM. FOR VARIOUS MATERIALS.

Density in Lines per Square Centimeter.	Magnetomotive Force in Ampere-turns per Centimeter of Length.		
	Forgings and Sheets of Wrought Iron or Steel.	Cast Steel.	Cast Iron.
1000	—	2	3
2000	0.6	3	7
3000	1.0	4	11
4000	1.5	5	17
5000	1.9	6	24
6000	2.3	7	33
7000	2.7	7	46
8000	3.0	8	62
9000	3.6	9	90
10 000	4.7	10	150
11 000	5.8	12	
12 000	7.0	15	
13 000	9.0	20	
14 000	12	27	
15 000	17	37	
16 000	25	50	
17 000	55	80	
18 000	95	130	
19 000	190		
20 000	300		
21 000	470		
22 000	670		
23 000	980		
24 000	1500		

Estimation of Component mmf. With these data we can estimate the mmf. required for each part of the magnetic circuit. Thus since the tooth density is 17 100 and since the core is built up of sheet steel, there will be required a mmf. of 57 ampere-turns (ats.) per cm. of length. The length of

the tooth is 5.7 cm., consequently the total mmf. required for the teeth is

$$5.7 \times 57 = 325 \text{ ats.}$$

Similar calculations may be made for the other parts. It is convenient to arrange these calculations in some such tabular form as the following:

Designation of the Part of the Magnetic Circuit.	Density in Lines per Square Centimeter.	Mmf. in Ampere-turns per Centimeter.	Length in Centimeters.	Required Mmf.
Teeth	17 100	57	5.7	330 ats.
Magnet core	17 500	100	20	2000 "
Magnet yoke	12 000	15	21	320 "
Stator core	10 000	4.7	41	190 "
Total mmf. for these four parts =				2840 ats.

The mmf. of Armature Interference. There is another matter to be dealt with before we can proceed with the air-gap calculations. This relates to the mmf. of the armature winding and is of importance. At load it interferes with the operation of the mmf. emanating from the field spools. We have arranged for a winding with 120 slots and 10 conductors per slot. Thus we have a total of $10 \times 120 = 1200$ armature conductors, or

$$\frac{1200}{2} = 600 \text{ turns.}$$

This comes to:

$$\frac{600}{3} = 200 \text{ turns per phase}$$

and

$$\frac{200}{8} = 25 \text{ turns per pole per phase.}$$

I, the full-load current, is equal to 120 amperes. Therefore we have $25 \times 120 = 3000$ ats. per pole per phase. This is termed the *armature strength*. Many designers resort to theoretical reasoning in ascertaining from the mmf. of one phase of the armature winding, the resultant mmf. exerted by the three phases. But in practice, the distribution of the stator and rotor windings, the ratio of the pole arc to the pitch, and other details of the design,

so complicate the matter as to considerably invalidate any theoretical deductions. But working backward from experimental observations it has been ascertained that, independently of the various relative dispositions of the windings and of other factors of the design, results consistent with practice may be arrived at by taking as the resultant mmf. of the three phases, 2.4 times the mmf. of each phase. Consequently, for our design we have, at full load, an armature mmf. of

$$2.4 \times 3000 = 7200 \text{ ats.}$$

It is only at zero power-factor that these armature ats. have the same axis as the field ampere-turns. If, when the power-factor of the external load is zero, the output is 120 amperes, then, if the current is lagging, the resultant mmf. acting to send flux round the magnetic circuit is obtained by subtracting 7200 ats. from the excitation on each field pole. If each field coil provided a mmf. of 15 000 ats., then the resultant mmf. would, for this lagging load of 120 amperes, be

$$15\,000 - 7200 = 7800 \text{ ats.}$$

If the 120 amperes were leading and if the power-factor were zero, then the resultant mmf. would be

$$15\,000 + 7200 = 22\,200 \text{ ats.}$$

For 120 amperes at other than zero power-factor, the armature mmf. does not affect the resultant mmf. to so great an extent. In a later section, we shall deal with a method of determining the extent of the influence of the armature mmf. when the power-factor is other than zero.

Even at this stage it is very evident from the phenomena that have been considered, that the armature mmf. will at heavy loads, and especially at overloads, exert a less disturbing influence the greater the mmf. provided on the field spools, and that for a given all-around quality of pressure regulation, the higher the armature mmf., the higher should be the field mmf. A rule usually leading to suitable pressure regulation, at usual power-factors, is to employ in each field spool a mmf. equal to *twice* the armature strength. A lower ratio is often employed; indeed it is often impracticable to find room for field spools supplying so high a mmf. But let

us endeavor to adhere to this ratio in the case of our example. Thus our field mmf. should be

$$2 \times 7200 = 14\,400 \text{ ats.}$$

But we have seen that the iron parts of our magnetic circuit only require a total mmf. of 2840 ats. How then can we employ a total mmf. of 14 400 ats. and not obtain through the armature winding, a greater flux than the 31.3 megalines which we have found to correspond to the required pressure of 12 000 volts (6950 volts per phase)?

We can so design the air-gap as regards density and length, as to use up the remaining 14 400 − 2840 = 11 560 ats., in overcoming its magnetic reluctance.

The Estimation of the Air-gap Density. First let us estimate the air-gap density. We have

$$\tau = 70 \text{ cm.}$$

$$\text{Pole arc} = 0.60 \times 70 = 42 \text{ cm.}$$

$$\text{Length of the pole shoe parallel to the shaft} = 114 \text{ cm.}$$

$$\text{Area of pole face} = 42 \times 114 = 4800 \text{ sq.cm.}$$

$$\text{Density in pole face} = \frac{31\,300\,000}{4800} = 6520 \text{ lines per sq.cm.}$$

Many designers employ complicated methods for estimating the density in the air-gap. These methods involve introducing "spreading coefficients" to allow for the flaring of the lines after they have emerged from the pole face. They also involve calculations of the area and density of the flux where it enters the armature surface. The author's own custom, in the case of alternators, is to usually take the air-gap density as equal to the pole-face density, though it is quite practicable afterward to use one's judgment in taking a somewhat higher or lower value according as the other conditions indicate that the pole-face density would be lower or higher than the air-gap density. In this machine, we need make no corrections of this sort, but may take the air-gap density as 6520 lines per sq.cm. This will require a mmf. of

$$\frac{10}{4\pi} \times 6520 = 5200 \text{ ats.}$$

per cm. of radial depth of the air-gap.

The Radial Depth of the Air-gap. Evidently the air-gap should have a radial depth of:

$$\frac{11\,560}{5200} = 2.22 \text{ cm.} = 22.2 \text{ mm.}$$

Since such calculations are necessarily only very rough, we shall do well to modify this and make the air-gap only 18 mm. deep. If, when the machine is tested, we find it desirable, we can increase the air-gap by turning down the rotor to a slightly smaller diameter.

The radial depth of the air-gap, in mm., may be denoted by $\triangle$. For our machine

$$\triangle = 18.$$

Revised Tabulation of Leading Diameters. Having now determined upon a value for the radial depth of the air-gap, let us again tabulate the various diameters in our machine:

External diameter of stator core	2300 mm.
Diameter at bottom of stator slots	1894 mm.
Internal diameter of stator (D)	1780 mm.
External diameter of rotor ($D-2\Delta$) [Revised from earlier table]	1744 mm.
Diameter to bottom of pole shoes	1580 mm.
External diameter of magnet yoke	1180 mm.
Internal diameter of magnet yoke	928 mm.

Saturation Curves at No Load. The next step is to work out data from which we can construct a no-load saturation curve for our machine, i.e., a curve in which ordinates will indicate the pressure per phase, in volts, and abscissae will indicate magneto-motive force per field spool in ats. We have previously obtained one point on this curve, namely the point for which we have found ordinate, 6950 volts; abscissa, 14 400 ats. The process can be considerably abbreviated in obtaining further points. Let us work out the mmf. required for 6000 volts, and for 7500 volts. With these three points we shall be able to construct the no-load saturation curve.

For the air-gap mmf. we obtain the desired results by simple proportion. Thus

$$\text{For 6000 volts: Air-gap ats.} = \frac{6000}{6950} \times 11\,560 = 10\,000 \text{ ats.}$$

$$\text{For 7500 volts: Air-gap ats.} = \frac{7500}{6950} \times 11\,560 = 12\,500 \text{ ats.}$$

But for the *iron* parts it is necessary first to obtain the flux density by simple proportion and then from Table 8, on page 33, obtain the corresponding values of the mmf. Thus for the teeth we have:

For 6950 volts: mmf. = 57 ats. per cm. (from p. 34).

$$\text{For 6000 volts: Density} = \frac{6000}{6950} \times 17\,100 = 14\,800 \text{ lines per sq.cm.}$$

Corresponding mmf. = 16 ats. per cm.

$$\text{For 7500 volts: Density} = \frac{7500}{6950} \times 17\,100 = 18\,400 \text{ lines per sq.cm.}$$

Corresponding mmf. = 125 ats. per cm.
Since the length is 5.7 cm., we have:
Total mmf. for teeth:

For 6000 volts: $16 \times 5.7 = 90$ ats.
For 6950 volts: $57 \times 5.7 = 330$ ats.
For 7500 volts: $125 \times 5.7 = 720$ ats.

In the same way we may estimate the corresponding values for magnet core, magnet yoke, and stator core. It is needless to record the steps in these calculations. The results are brought together in the following table:

	6000 Volts.	6950 Volts.	7500 Volts.
Air-gap	10 000 ats.	11 560 ats.	12 500 ats.
Teeth	90 "	330 "	720 "
Magnet core	770 "	2000 "	5300 "
Magnet yoke	200 "	320 "	420 "
Stator core	140 "	190 "	230 "
Total mmf.	11 200 ats.	14 400 ats.	19 170 ats.

The no-load saturation curve in Fig. 23 has been plotted from these results.

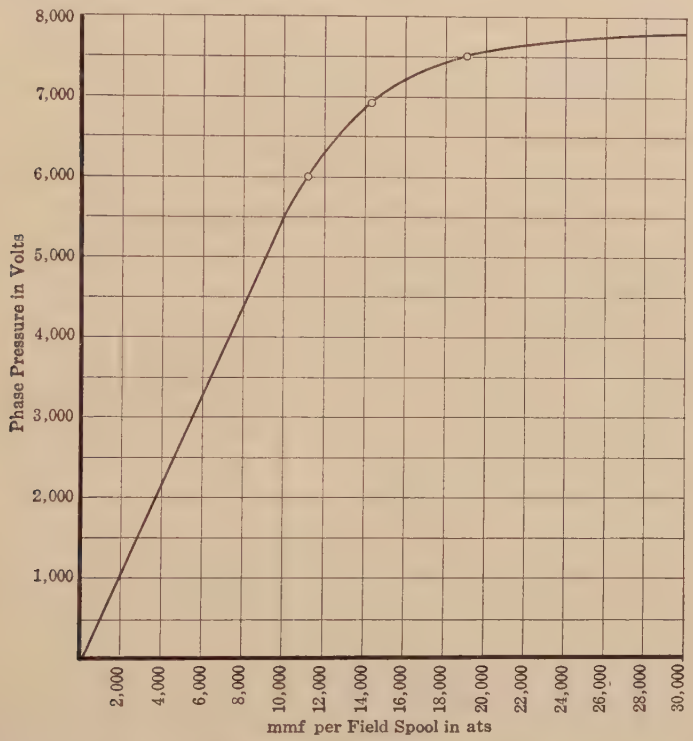


FIG. 23.—No-load Saturation Curve of 2500-kva. Generator with an Air-gap of 18 mm.

THE PRESSURE REGULATION

The subject of this section is one to which much study and discussion have been devoted. Nevertheless we still find wide differences of opinion with reference to the problems involved in the estimation of the excitation required in any given case and for any given conditions of operation.

The method which will be described has the merit of brevity combined with *at least* as much exactness, so far as regards the results obtained, as can be shown to be possessed by any other method. The chief defect in the method relates to the theoretical indefensibility from the *quantitative* standpoint of certain steps

in the calculations. Since, however, the occurrences assumed to take place are *qualitatively* in accordance with the facts, it is believed that the admitted defect is of minor importance.

Before proceeding to explain the method in applying it to our 2500-kva. design, let us bring together the leading data which we have now worked out. This is done in the following specification:

SUMMARY OF THE NORMAL RATING OF THE DESIGN

Number of poles	8
Output at full load in kva	2500
Corresponding power-factor of external load	0.90
Corresponding output in kw	2250
Speed in r.p.m	375
Periodicity in cycles per second	25
Terminal pressure in volts	12 000
Number of phases	3
Connection of phases	Y
Pressure per phase $\left(\frac{12\,000}{\sqrt{3}}\right)$	6950

THE LEADING DATA OF THE DESIGN

External diameter of stator core	2300 mm.
Diameter at bottom of stator slots	1894 mm.
Internal diameter of stator (D)	1780 mm.
External diameter of rotor	1744 mm.
Diameter at bottom of pole shoes	1580 mm.
External diameter of magnet yoke	1180 mm.
Internal diameter of magnet yoke	928 mm.
Polar pitch (τ)	700 mm.
Gross core length (λg)	1180 mm.
Number of vertical ventilating ducts	20
Width each duct	15 mm.
Per cent insulation between laminations	10 per cent
Net core length $(1180-300)\times 0.90$ (λn)	792 mm.
Length pole shoe parallel to shaft	1140 mm.
Pole arc	420 mm.
Area of pole face (114×42)	4790 sq.cm.
Extreme length magnet core parallel to shaft	860 mm.
Extreme width magnet core	260 mm.
Area of cross-section of magnet core	2080 sq.cm.

Length of yoke parallel to shaft	1200 mm.
Radial depth of yoke	126 mm.
Cross-section of yoke	1510 sq.cm.
Number of stator slots	120
Number stator slots per pole per phase $\left(\frac{120}{8\times3}\right)$	5
Depth slot	57 mm.
Width slot	24 mm.
Width slot opening	12 mm.

Sketches of the design have already been given in Figs. 20 and 21. These preliminary sketches show wide-open slots. Let us, however, employ slots with an opening of only 12 mm. in accordance with the above tabulated specification.

The stator winding consists of 10 conductors per slot. The bare dimensions of each conductor are 12 mm.×3.3 mm., and the 10 conductors are arranged one above the other as already indicated in Fig. 1 on page 11. The mean length of one armature turn is 656 cm. The winding is of the type which we have termed a "half-coiled" winding. That is to say, only half the field poles have opposite to them, armature coils belonging to any one phase. A diagram of one phase of a typical half-coiled winding for a 6-pole machine has been given in Fig. 15 on p. 20. and is seen to have $\left(\frac{6}{2}\right)$ = three coils per phase. Since our 2500 kva. machine has 8 poles, there are four coils in each phase. Each side of each coil comprises the contents of five adjacent slots. Since each slot contains 10 conductors, there are (5×10=) 50 turns per coil, and consequently (4×50=) 200 turns in series per phase. Denoting by T the number of turns in series per phase we have

$$T=200.$$

In Fig. 24 is given a winding diagram for one of the three phases, and in Fig. 25, is given a winding diagram containing all three phases.

For a pressure of $\left(\frac{12\,000}{\sqrt{3}}\right)$ = 6950 volts per phase on open circuit, the armature flux per pole (denoted by M), is obtained as follows:

$$M=\frac{6950}{0.0444\times200\times25}=31.3 \text{ megalines.}$$

We may take the leakage factor as 1.15. Consequently the flux in the magnet core and yoke is:

$$1.15 \times 31.3 = 36.0 \text{ megalines.}$$

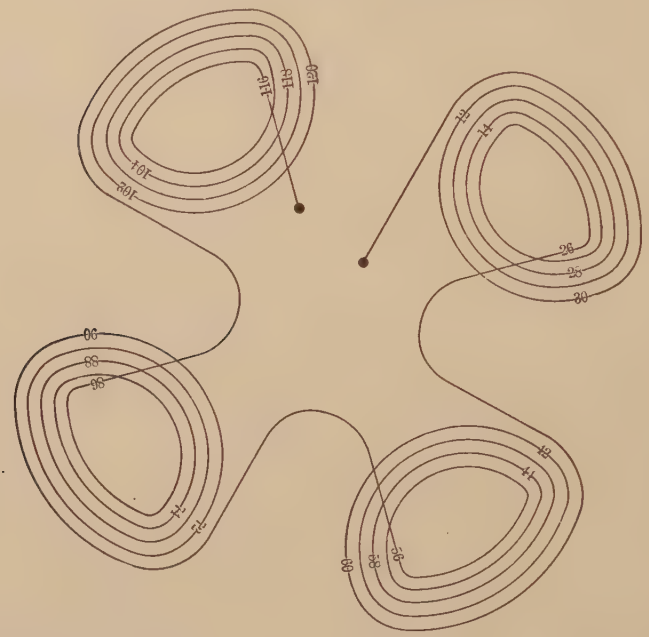


FIG. 24.—Winding Diagram for one of the Three Phases of the 2500-kva. three-phase Alternator.

The estimation of the no-load saturation curve has previovsly been carried out in earlier sections. A summary of the component and resultant magnetomotive forces (mmf.) for 6000, 6950, and 7500 volts, is given in the following table:

	6000 Volts.	6950 Volts.	7500 Volts.
Air-gap	10 000 ats.	11 560 ats.	12 500 ats.
Teeth	90 "	330 "	720 "
Magnet core	770 "	2000 "	5300 "
Magnet yoke	200 "	320 "	420 "
Stator core	140 "	190 "	230 "
Total mmf	11 200 ats.	14 400 ats.	19 170 ats.

A no-load saturation curve passing through these three points has been given in Fig. 23, on page 39.

The Armature Interfering mmf. The current per phase at rated load of 2500 kva. is:

$$I = \frac{2\,500\,000}{3 \times 6950} = 120.$$

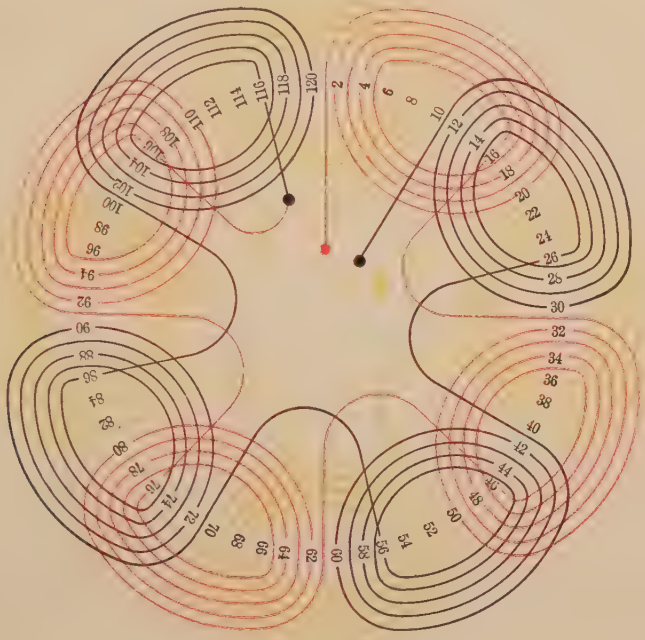


FIG. 25.—Complete Winding Diagram for all Three Phases of the 2500-kva. Generator.

We have seen that there are 200 turns in series in each phase. Consequently there are:

$$\frac{200}{8} = 25 \text{ turns per pole per phase}$$

and

$$25 \times 120 = 3000 \text{ (rms.) ats. per pole per phase.}$$

It has already been stated that many designers resort to theoretical reasoning in ascertaining from the mmf. of one phase,

the resultant mmf. exerted by the three phases. But, in practice, the distribution of the stator and rotor windings, the ratio of the pole arc to the pitch, and other details of the design, so complicate the matter as considerably to invalidate any theoretical deductions. But working backward from a very large collection of experimental observations, the conclusion is reached that independently of the various relative dispositions of the winding and of other features of the design, results consistent with practice are obtained by taking the resultant mmf. of the three phases as equal to:

2.4 times the mmf. of each phase.

Consequently for our design, we have, at full load, an armature mmf. of:

$$2.4 \times 3000 = 7200 \text{ ats.}$$

It is only at zero power-factor that these armature ats. have the same axis as the field ampere-turns. If, when the power-factor of the external load is zero, the output is 120 amperes, then, if the current is lagging, the resultant mmf. acting to send flux round the magnetic circuit is obtained by *subtracting* 7200 ats. (the armature mmf.) *from* the excitation on each field pole. If, with the power-factor again equal to zero, the current is leading, then the resultant mmf. acting to send flux round the magnetic circuit is obtained by *adding* 7200 ats. (the armature mmf.) *to* the excitation on each field pole. For this same current of 120 amperes, but at other than zero power-factor, the armature mmf. does not affect the resultant mmf. to so great an extent.

Later we shall consider a method of determining the extent of the influence of the armature mmf. when the power-factor is other than zero. It follows as a consequence of the preceding explanations that the armature mmf. will exert a less disturbing influence on the terminal pressure the greater the mmf. provided on the field spools, and that for a given required closeness of pressure regulation the higher the armature mmf., the higher must also be the field mmf.

The modern conception of preferable conditions is not based on such close inherent pressure regulation as was formerly considered desirable. The alteration in conceptions in this respect

does not, however, decrease the importance of having at our disposal means for accurately estimating the excitation required under various conditions of load as regards pressure, power-factor and amount.

The Field Excitation Required with Various Loads. The required excitation is chiefly dependent upon three factors:

1. The no-load saturation curve of the machine;
2. The armature strength in ats. per pole;
3. The inductance of the armature winding.

In our 2500-kva three-phase machine, the armature strength at rated load is equal to 7200 ats.

The Position of the Axis of the Armature mmf. If the armature winding had no inductance, then for an external load of unity power-factor, the axis of the armature magnetomotive force would be situated just midway between two adjacent poles; that is to say, there would be no direct demagnetization. At the other extreme, namely for the same current output at zero power-factor, the axis of armature demagnetization would correspond with the field axis. The two cases are illustrated diagrammatically in Figs. 26 and 27. In our machine, when loaded with full-load current of 120 amperes *at zero power-factor*, the demagnetization would amount to 7200 ats. and this demagnetization could only be offset by providing 7200 ats. on each field pole. For power-factors between 1 and 0, the axis of armature demagnetization would be intermediate, as indicated diagrammatically in Fig. 28.

But we are not concerned with imaginary alternators with zero-inductance armature windings, but with actual alternators. In actual alternators, the armature windings have considerable inductance. At this stage we wish to determine the inductance of the armature windings of our 2500-kva alternator.

The Inductance of a 1-turn Coil. Let us first consider a single turn of the armature winding before it is put into place in the stator slots. If we were to send one ampere of continuous electricity through this turn, how many magnetic lines would be occasioned? If the conductor were large enough to practically fill the entire slot, then with the dimensions employed in modern alternators, the general order of magnitude of the flux occasioned may be ascertained on the basis that some 0.3 to 0.9 of a line would be

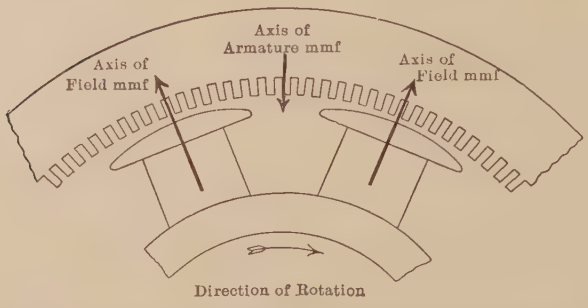


FIG. 26—Diagrammatic Representation of Relative Positions of Axes of Field mmf. and Armature mmf. for a Load of Unity Power-factor Neglecting Armature Inductance.

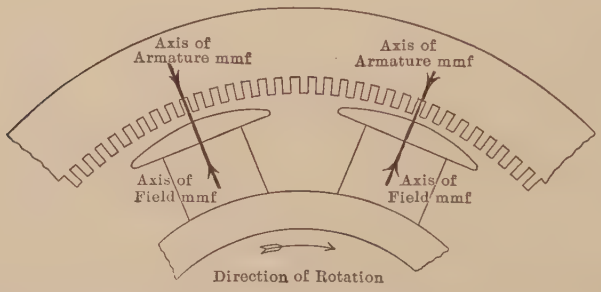


FIG. 27.—Diagrammatic Representation of Relative Positions of Axes of Field mmf. and Armature mmf. for a Load of Zero Power-factor.

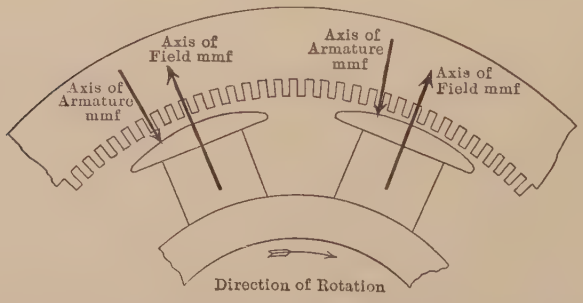


FIG. 28.—Diagrammatic Representation of Relative Positions of Axes of Field mmf. and Armature mmf. for a Load of Intermediate Power-factor.

linked with every centimeter of length of the turn. Taking the mean value of 0.6 line per cm., then since in our design the length of a turn is 656 cm., the flux occasioned by 1 amp. of continuous electricity is

$$656 \times 0.6 = 394 \text{ lines.}$$

The inductance (expressed in henrys) of a 1-turn coil is equal to 10^{-8} times the number of lines linked with the turn when 1 amp. of continuous electricity is flowing through the turn. Consequently the inductance is, in this case, equal to

$$10^{-8} \times 394 = 0.00000394 \text{ henry.}$$

The Inductance of a Coil with More than One Turn. The inductance of any coil is equal to the product (when 1 amp. of continuous electricity is flowing through the coil), of the flux linked with the coil and the number of turns in the coil. This definition is framed on the assumption that the entire flux is linked with the entire number of turns. Where this is not the case, appropriate factors must be employed in order to arrive at the correct result.

In a two-turn coil, the mmf. is, when a current of 1 amp. of continuous electricity is flowing through the coil, twice as great as in a one-turn coil of the same dimensions. Consequently for a magnetic circuit of air, the *flux* will also be twice as great, since in air the flux is directly proportional to the mmf. occasioning it. But since this doubled flux is linked with double the number of turns, the total linkage of flux and turns is *four* times as great. In other words, the inductance increases as the *square* of the number of turns.

In Fig. 24, it has been shown that the winding of any one phase of our eight-pole machine is composed of four coils in series. Let us first consider one of those four coils. Each side comprises the contents of five slots. Since there are ten conductors per slot, we see that we are dealing with a fifty-turn coil. On the sufficient assumption that the incomplete linkage of flux and turns is provided for by calculating from the basis of only 0.5 line per centimeter of length, instead of from the value of 0.6 line per centi-

meter of length which we employed when dealing with the one-turn coil, we obtain for the inductance the value:

$$50^2 \times \frac{0.5}{0.6} \times 0.00000394 = 0.0082 \text{ henry.}$$

The Inductance and Reactance of One Phase. The winding of one phase comprises four such coils in series, and consequently we have:

$$\text{Inductance per phase} = 4 \times 0.0082 = 0.0328 \text{ henry}$$

The reactance is obtained from the formula:

$$\text{Reactance (in ohms)} = 2\pi \sim l.$$

where the periodicity in cycles per second is denoted by $\sim$ and the inductance in henrys by l. We consequently have:

$$\text{Reactance per phase} = 6.28 \times 25 \times 0.0328 = 5.15 \text{ ohms.}$$

The Reactance Voltage per Phase. For our machine the full-load current per phase is 120 amperes. Consequently when carrying full-load current we have:

$$\text{Reactance voltage per phase} = 120 \times 5.15 = 618 \text{ volts.}$$

The Inductance and Reactance of Slot-embedded Windings. But up to this point we have considered that throughout their length the windings *are surrounded by air*. In reality the windings are embedded in slots for a certain portion of their length. For this embedded portion of their length, the flux, in lines per centimeter of length, set up in a one-turn coil when one ampere of continuous electricity flows through it, is considerably greater than for those portions of the coil which are surrounded by air. Suitable values may be obtained from Table 9:

TABLE 9. DATA FOR ESTIMATING THE INDUCTANCE OF THE EMBEDDED LENGTH.

	No. of Lines per cm.
Concentrated windings in wide-open, straight-sided slots	3 to 6
Thoroughly distributed windings in wide-open, straight-sided slots	1.5 to 3
Concentrated windings in completely-closed slots	7 to 14
Thoroughly-distributed windings in completely-closed slots	3 to 6
Partly distributed windings in semi-closed slots	about 5

In order to illustrate the sense in which the terms "concentrated," "thoroughly-distributed" and "partly-distributed" windings are employed in the above table, the three winding diagrams in Figs. 29, 30 and 31 have been prepared. Evidently for the windings of any one phase of our machine, the value of 5 lines per cm. of embedded length is sufficiently representative.

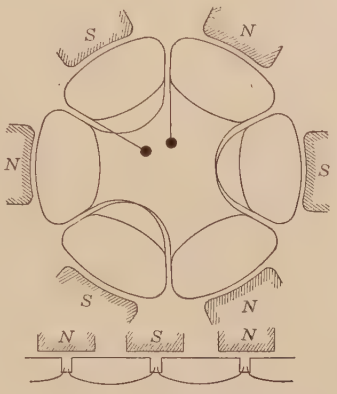


FIG. 29.—Concentrated Winding.

The embedded portion of the length of a turn is equal to twice the net core length. For our machine we have:

Embedded length $= 2 \times 79 = 158$ cm.;
Mean length of a turn $= 656$ cm.
"Free" length (i.e., the portion in air) $= 656 - 158 = 498$ cm.

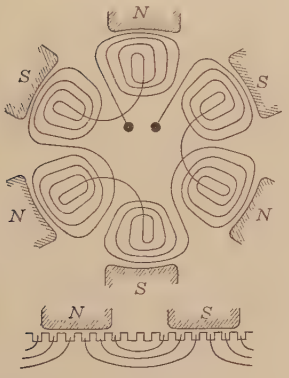


FIG. 30.
Thoroughly Distributed Winding.

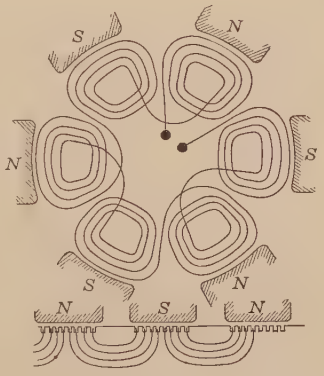


FIG. 31.
Partly Distributed Winding.

We have calculated the inductance which our coil would have if the entire 656 cm. of its length were surrounded by air (i.e., were "free" length). We can now readily obtain the value of

that part of the inductance which is associated with the *actual* "free" length. It amounts to:

$$\frac{498}{656}\times 0.0328 = 0.0248 \text{ henry.}$$

The inductance of the "embedded" length is

$$\frac{158}{656}\times\frac{5.0}{0.5}\times 0.0328 = 0.0790 \text{ henry.}$$

The total inductance per phase is

$$0.0248 + 0.0790 = 0.104 \text{ henry.}$$

It is interesting to note that

$$\frac{0.0248}{0.104}\times 100 = 23.8 \text{ per cent}$$

of the total inductance, is, in the case of this particular machine, associated with the end connections.

Our estimate of the inductance has been so seriously interrupted by explanatory text that it is desirable to set it forth again in a more orderly form, and taking each step in logical order:

Mean length of turn	656 cm.
"Free" length	498 cm.
"Embedded" length	158 cm.

Flux per ampere-turn per centimeter of length { 0.5 line for "free" length, 5.0 lines for "embedded" length. }

Flux per ampere-turn { 249 lines for "free" length, 790 lines for "embedded" length. }

Total flux per ampere-turn (= 249 + 790 =) 1040 lines.

Number of turns in one phase per pair of poles (i.e., per coil) 50

Inductance of one coil ($1040 \times 50^2 \times 10^{-8} =$) ..... 0.0260 henry
Number of coils (also pairs of poles) per phase..... 4
Inductance of one phase ($4 \times 0.0260 =$) ..... 0.104 henry
Reactance of one phase at 25 cycles ($6.28 \times 25 \times 0.104 =$) 16.3 ohms
Reactance voltage of one phase at 25 cycles and 120 amperes ($120 \times 16.3 =$) ..... 1960 volts

Physical Conditions Corresponding to this Value of the Reactance Voltage. This value of 1960 volts for the reactance voltage, is of the order of the value which we should obtain experimentally under the following conditions:

Twenty-five-cycle current is sent into the stator windings from some external source, while the rotor (unexcited), is, by means of a motor, driven at the slowest speed consistent with steady indications of the current flowing into the three branches of the stator windings. Under these conditions, some 1960 volts per phase would be found to be necessary in order to send 120 amperes into each of the three windings.

Theta (θ) and Its Significance. The value of the reactance voltage thus determined, enables us to ascertain the angular distance from mid-pole-face position at which the current in the stator windings passes through its crest value.

Let this angle be denoted by θ. For a load of unity power-factor, θ is the angle whose tangent is equal to the reactance voltage divided by the phase voltage. Thus we have:

$$\theta = \tan^{-1} \frac{\text{reactance voltage}}{\text{phase voltage}}.$$

The conception of θ may possibly be made clearer by stating that it represents the angular distance by which the center of a group of conductors belonging to one phase has traversed beyond mid-pole-face position when the current in these conductors reaches its crest value.

Theta at Unity Power-factor. For our example we have (for unity power-factor):

$$\theta = \tan^{-1} \frac{1960}{6950} = \tan^{-1} 0.282 = 15.9°.*$$

* In making calculations of the kind explained in this Chapter, the Table of sines, cosines and tangents in Appendix III. will be found useful.

The diagram is shown in Fig. 32. Strictly speaking, we ought to take into account in the diagram, the IR drop in the armature.

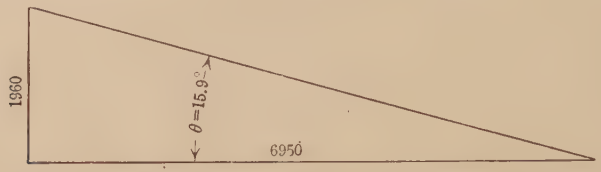


FIG. 32.—Diagram Relating to the Explanation of the Nature and Significance of the Angle Theta (θ).

The resistance per phase (at 60° Cent.), is 0.685 ohm. Consequently for the full-load current of 120 amperes, we have:

$$IR \text{ drop} = 120 \times 0.685 = 82 \text{ volts.}$$

The corrected diagram (taking into account the IR drop), is shown in Fig. 33. In this diagram we have:

$$\theta = \tan^{-1}\frac{1960}{6950+82} = \tan^{-1}\frac{1960}{7032} = \tan^{-1} 0.278 = 15.8°.$$

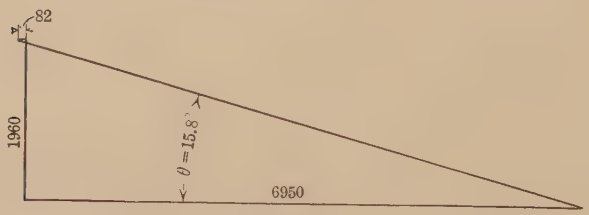


FIG. 33.—More Exact Diagram for Obtaining θ.

Relation between Theta and the Armature Interference. The armature demagnetization for any value of θ is obtained by multiplying the armature strength by sin θ.

For 120 amperes the armature strength is:

$$7200 \text{ ats.}$$

We also have:

$$\sin \theta = \sin 15.8° = 0.270.$$

Under these conditions (120 amperes output at unity power-factor of the external load), the armature mmf. is equal to:

$$0.270 \times 7200 = 1940 \text{ ats.}$$

The Hypothenuse of the θ-Triangle Has no Physical Existence. It is desirable to lay strong emphasis on the fact that the vector sum of 7032 volts and 1960 volts does not represent an actually-existing internal pressure corresponding to an actual flux of magnetic lines. The quantity which, earlier in this chapter, has been termed the "reactance voltage", is made up of two parts, associated respectively with the "embedded" length and with the "free" length. While the portion associated with the "embedded" length manifests itself in distortion of the magnetic flux, the portion associated with the "free" length acts in the same manner as would an equal inductance located in an independent inductance coil connected in series with a non-inductive alternator. (More strictly, it is only that portion of the "free" length which is associated with the end connections which should be thus considered, and the portion associated with the ventilating ducts should be placed in a different category. But in practice the small margin provided by taking the *entire* "free" length, is desirable.)

We have seen that the inductance of the "free" length of the windings of our 2500-kva. machine is 23.8 per cent of the total inductance:

$$0.238 \times 1960 = 466 \text{ volts.}$$

The True Internal Pressure and Its Components. The three components of the total internal pressure of our machine, when the external load is 120 amperes at unity power-factor, are, per phase:

Phase pressure	6950 volts
IR drop	82 "
Reactance drop	466 "

When these are correctly combined, as shown in Fig. 34, the internal pressure is ascertained to be:

$$\sqrt{7032^2+466^2}=7050 \text{ volts.}$$

The influence of the reactance voltage is thus (for these particular conditions of load), practically negligible, so far as concerns occasioning an internal pressure appreciably exceeding the resultant of the terminal pressure and the IR drop.

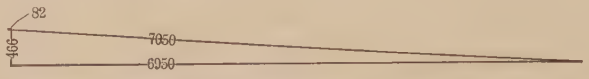


FIG. 34.—Pressure Diagram Corresponding to 6950 Terminal Volts and 120 Amperes at Unity Power-factor.

Total mmf. Required at Full Load and Unity Power-factor. From the no-load saturation curve in Fig. 23 on p. 39, we see that we require:

15200 ats.

to overcome the reluctance of the magnetic circuit when the internal pressure is:

7050 volts.

We require further:

1940 ats.

to offset the armature demagnetization for these conditions of load (120 amperes at unity power-factor). Consequently we require a total mmf. per field spool, of:

$$15\ 200+1940=17\ 140 \text{ ats.}$$

That is to say, for full-load conditions (6950 volts per phase and 120 amperes at unity power-factor), we require an excitation of:

17 140 ats.

The Inherent Regulation at Unity Power-factor. We can now ascertain from the saturation curve the value to which the

pressure will rise, when, maintaining constant this excitation of 17 140 ats., we decrease the load to zero. We find the value of the pressure to be:

7350 volts.

Thus the pressure rise occurring when the load is decreased to zero, is:

$$\left(\frac{7350-6950}{6950}\times 100=\right)5.8 \text{ per cent.}$$

This is expressed by stating that at unity power-factor the inherent regulation is:

5.8 per cent.

ESTIMATION OF SATURATION CURVE FOR UNITY POWER FACTOR AND 120 AMPERES

Let us now proceed to calculate values from which we can plot a load saturation curve extending from a pressure of 0 volts up to a pressure of 7500 volts for an external load of 120 amperes at unity power-factor. We already have one point; namely:

17 140 ats. for 6950 volts.

For this unity power-factor, 120-ampere saturation curve, the terminal pressure will be varied from 0 up to a phase pressure of—say—7500 volts while the current is held constant at 120 amperes. The diagrams for obtaining θ for 7500, 5000, 2500 and 0 volts are shown in Fig. 35. For these four cases we have:

1. $\theta=\tan^{-1}\frac{1960}{7582}=\tan^{-1}0.258=14.5°$ $\quad\sin 14.5°=0.250$

2. $\theta=\tan^{-1}\frac{1960}{5082}=\tan^{-1}0.386=21.1°$ $\quad\sin 21.1°=0.360$

3. $\theta=\tan^{-1}\frac{1960}{2582}=\tan^{-1}0.758=37.1°$ $\quad\sin 37.1°=0.605$

4. $\theta=\tan^{-1}\frac{1960}{82}=\tan^{-1}23.9=87.5°$ $\quad\sin 87.5°=0.999$

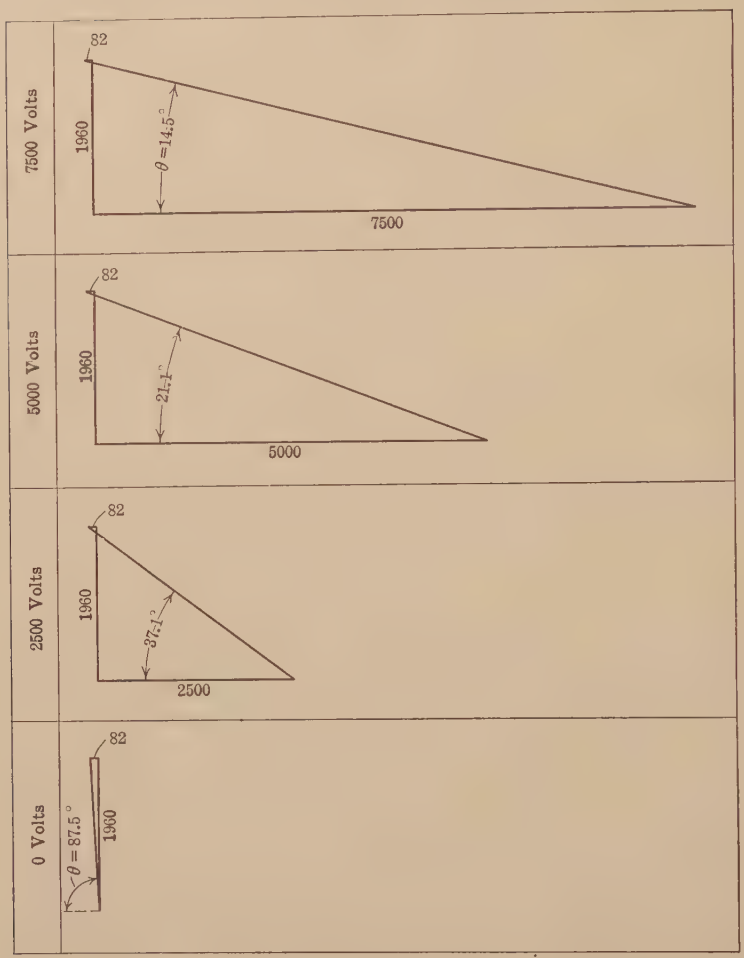


FIG. 35.—Theta Diagrams for 120 Amperes at Unity Power-factor.

Since the armature current is, in all four cases, 120 amperes, the armature strength remains 7200 ats. The armature demagnetization amounts, in the four cases, to:

1. $0.250 \times 7200 = 1800$ ats.;
2. $0.360 \times 7200 = 2590$ "
3. $0.605 \times 7200 = 4350$ "
4. $0.999 \times 7200 = 7200$ "

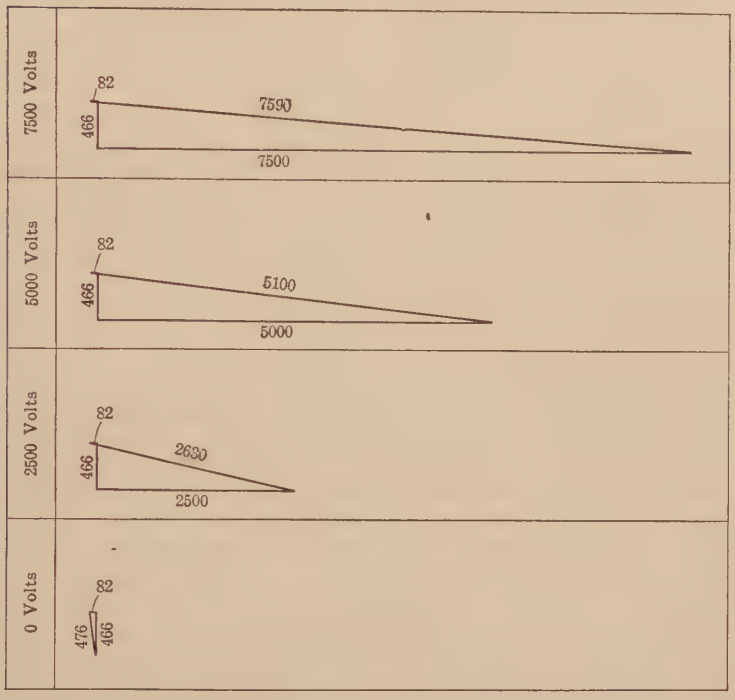


FIG. 36.—Pressure Diagrams for 120 Amperes at Unity Power-factor.

The Armature Reaction with Short-circuited Armature. It is interesting to note that in the last diagram in Fig. 35, i.e., in the diagram relating to zero terminal pressure (short-circuited armature) the angle θ is practically 90°. Consequently the armature reaction with short-circuited armature, is practically identical with the armature strength expressed in ampere-turns per pole.

The Required Field Excitation for Each Terminal Pressure. The field excitation at each pressure, comprises two components. The first of these components must be equal to the armature demagnetization (in order to neutralize it), and the second component must be of the right amount to drive the required flux through the magnetic circuit in opposition to its magnetic reluctance. This latter value may be obtained from the no-load saturation curve in Fig. 23 (on p. 39), and must correspond to the

four pressures obtained from the four diagrams in Fig. 36. These four pressures are:

1. $\sqrt{7582^2+466^2}=7590$ volts.
2. $\sqrt{5082^2+466^2}=5100$ "
3. $\sqrt{2582^2+466^2}=2630$ "
4. $\sqrt{82^2+466^2}=476$ "

The saturation ats. for these four pressures are found from Fig. 23 (on p. 39), to be as follows:

	Pressure	Sat. Ats
1.	7590	22 000
2.	5100	9 100
3.	2630	4 700
4.	476	820

The derivation of the total required mmf. is arranged below in tabular form:

	Terminal Pressure.	Saturation Ats.	Ats.for Offsetting Armature Demagnetization.	Total Required Ats.
1	7500	22 000	1800	23 800
2	5000	9100	2950	11 690
3	2500	4700	4350	9050
4	0	820	7200	8020

The unity-power-factor, 120-ampere, saturation curve thus derived, is plotted in Fig. 37, where the no-load saturation curve is also reproduced from Fig. 23.

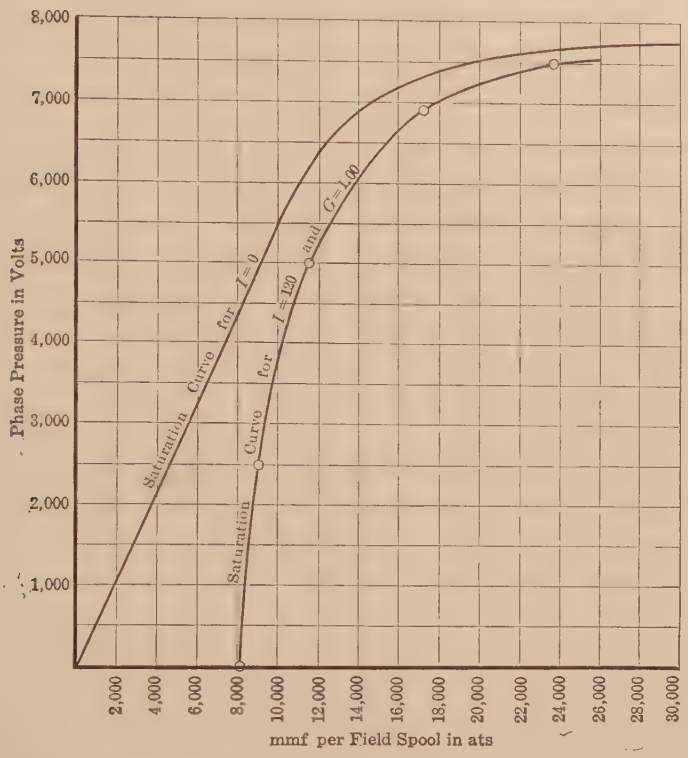


FIG. 37.—Saturation Curves for $I=120$ and $G=1.00$ and for $I=0$.

ESTIMATION OF SATURATION CURVE FOR UNITY POWER-FACTOR AND 240 AMPERES

Let us now construct a saturation curve for unity power-factor and 240 amperes output, i.e., for twice full-load current.

The diagrams of Figs. 35 and 36 are now replaced by those of Figs. 38 and 39.

The reactance voltage for the θ-diagrams of Fig. 38 is now twice as great as before, since the current is now 240 amperes in place of 120 amperes.

The reactance voltage is now:

$$2\times 1960 = 3920 \text{ volts.}$$

The IR drop is now:

$$2\times 82 = 164 \text{ volts.}$$

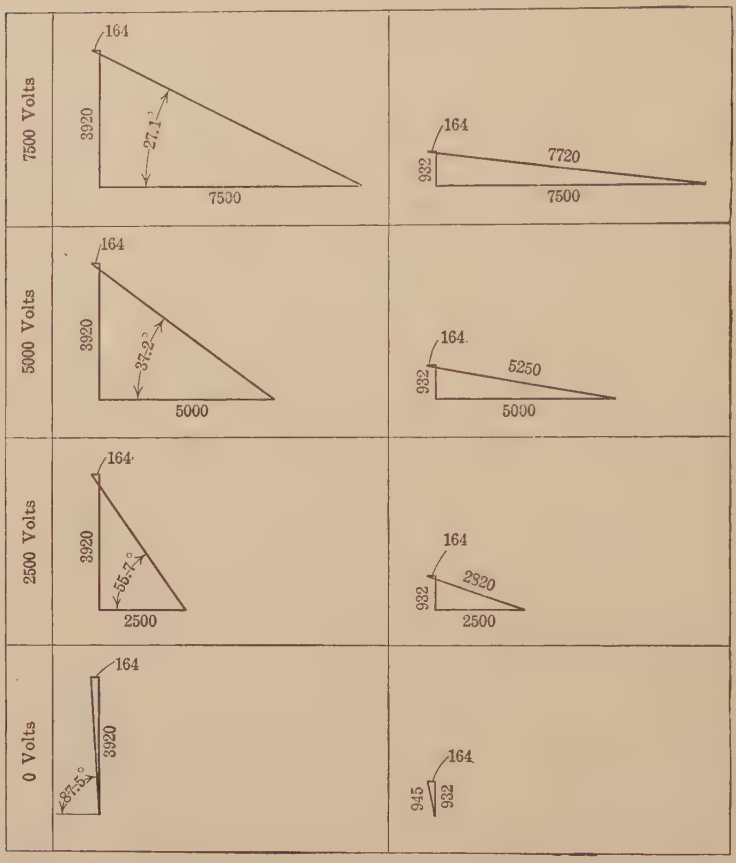


FIG. 38.—Theta Diagrams for $I=240$ and $G=1.00$.

FIG. 39.—Pressure Diagrams for $I=240$ and $G=1.00$.

Consequently:

1. $\theta=\tan^{-1}\frac{3920}{7664}=\tan^{-1}0.511=27.1°$ $\quad\sin 27.1°=0.456$

2. $\theta=\tan^{-1}\frac{3920}{5164}=\tan^{-1}0.760=37.2°$ $\quad\sin 37.2°=0.605$

3. $\theta=\tan^{-1}\frac{3920}{2664}=\tan^{-1}1.47\ =55.7°$ $\quad\sin 55.7°=0.826$

4. $\theta=\tan^{-1}\frac{3920}{164}=\tan^{-1}23.8\ =87.5°$ $\quad\sin 87.5°=0.999$

The armature strength is now;

$$2\times 7200 = 14\,400 \text{ ats.}$$

Consequently in the four cases, we now have for the armature demagnetizing ats.:

1. $0.456\times 14400 = 6580$ ats.
2. $0.605\times 14400 = 8700$ "
3. $0.826\times 14400 = 11900$ "
4. $0.999\times 14400 = 14400$ "

The internal inductance pressure is now:

$$2\times 466 = 932 \text{ volts.}$$

The four internal pressures and the corresponding saturation ats. are:

	Internal Pressures.	Sat. Ats.
1.	$\sqrt{7664^2+932^2} = 7720$	29 500
2.	$\sqrt{5164^2+932^2} = 5250$	9 350
3.	$\sqrt{2664^2+932^2} = 2820$	5 150
4.	$\sqrt{164^2+932^2} = 945$	1 700

The total required ats. are shown in the last column of the following tabulated calculation:

	Term. Pres.	Saturation Ats.	Ats. for Offsetting Armature Demagnetization.	Total Required Ats.
1	7500	29500	6580	36 080
2	5000	9350	8700	18 050
3	2500	5150	11900	17 050
4	0	1700	14400	16 100

The values in the last column are the basis for the unity power-factor, 240-ampere saturation curve shown in Fig. 40.

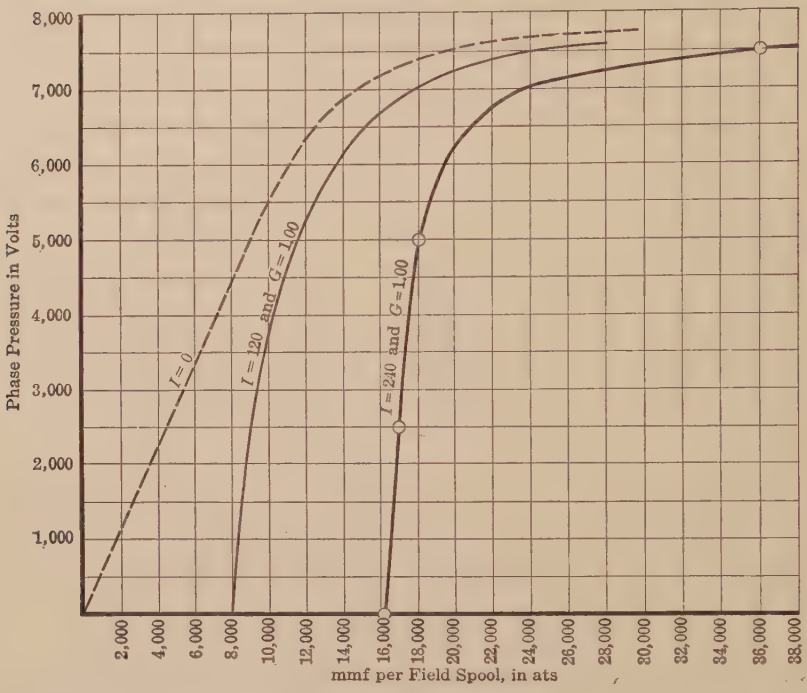


FIG. 40.—Saturation Curves for Various Values of I and for $G = 1.00$.

The unity power-factor, 120-ampere curve is also reproduced from Fig. 37 and the $I = 0$ curve from Fig. 23.

SATURATION CURVES FOR POWER-FACTORS OF LESS THAN UNITY

Let us now return to a load of 120 amperes, but let the power-factor of the load on the generator be 0.90. Let us estimate the required mmf. under these conditions, for terminal pressures of 7500, 5000, 2500, and 0 volts, and then, from these four results, let us plot a 0.90-power-factor, 120-ampere, saturation curve.

The angle θ, i.e., the angle by which the conductors have passed mid-pole-face position when carrying the crest current, is now obtained as follows:

When the power-factor of the external load is 0.90, the current lags behind the terminal pressure by 26.0°, since cos 26° = 0.90.

The θ-diagram for 120 amperes and a terminal pressure of 6950 volts, is now as shown in Fig. 41. The entire object of this diagram is to obtain the angle θ, i.e., the angle by which the con-

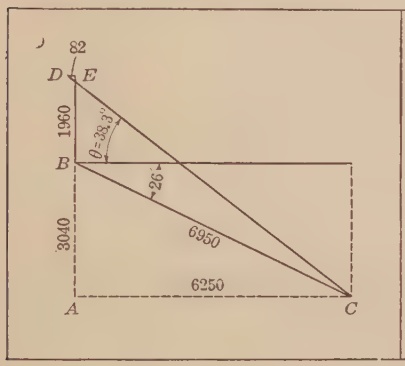


FIG. 41.—Theta Diagram for $I=120$ and $G=0.90$.

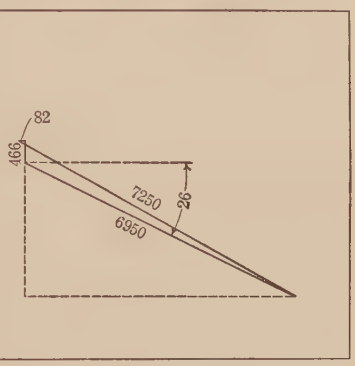


FIG. 42.—Pressure Diagram for $I=120$ and $G=0.90$.

ductors have passed mid-pole-face position when the current is at its crest value.

$$AB = BC \sin 26°$$

$$= 6950 \times 0.438 = 3040$$

$$AC = 0.90 \times 6950 = 6250$$

$$\tan \theta = \frac{AE}{AC+DE} = \frac{AB+BE}{AC+DE}$$

$$= \frac{3040+1960}{6250+82}$$

$$= \frac{5000}{6332}$$

$$= 0.790.$$

Therefore

$$\theta = 38.3°$$

$$\sin 38.3° = 0.620.$$

Therefore:

Armature demagnetizing ats. $= 0.620 \times 7200 = 4450$ ats.

The diagram for obtaining the internal pressure is shown in Fig. 42. By scaling off from this diagram, we find that the internal pressure is 7250 volts. From the no-load saturation curve we find:

$$\text{Saturation mmf. for 7250 volts} = 16\,700 \text{ ats.}$$

Thus the total required mmf. for a phase pressure of 6950 volts with a load of 120 amperes at a power-factor of 0.90, is:

$$4450 + 16\,700 = 21\,150 \text{ ats.}$$

For loads of other than unity power-factor, the most expeditious method of arriving at the results is usually that by graphical constructions. In the chart of Fig. 43 which relates to the graphical derivation of the saturation curve for 120 amperes at 0.90 power-factor, the diagrams in the right-hand column relate to the determination of the internal pressure. The first, second, third and fourth horizontal rows relate respectively to the diagrams for phase pressures of 7200, 5000, 2500 and 0 volts.

The left-hand vertical row of diagrams relates to the constructions for the determination of θ for these four terminal pressures.

From the internal-pressure diagrams in Figs. 42 and 43 and from the no-load saturation curve in Fig. 23 we find:

Phase Pressure.	Internal Pressure.	Saturation Ats.
0	472	1 000
2500	2800	5 000
5000	5290	9 300
6950	7250	16 700
7200	7500	19 200

From the θ diagrams we obtain the following results:

Phase Pressure.	Tan θ.	Angle θ.	Sin θ.	Ampere-turns Required to Offset Armature Demagnetization.
0	24	87.5°	0.999	7200
2500	1.35	53.5°	0.804	5780
5000	0.918	42.5°	0.675	4850
6950	0.790	38.3°	0.620	4450
7200	0.785	38.1°	0.617	4440

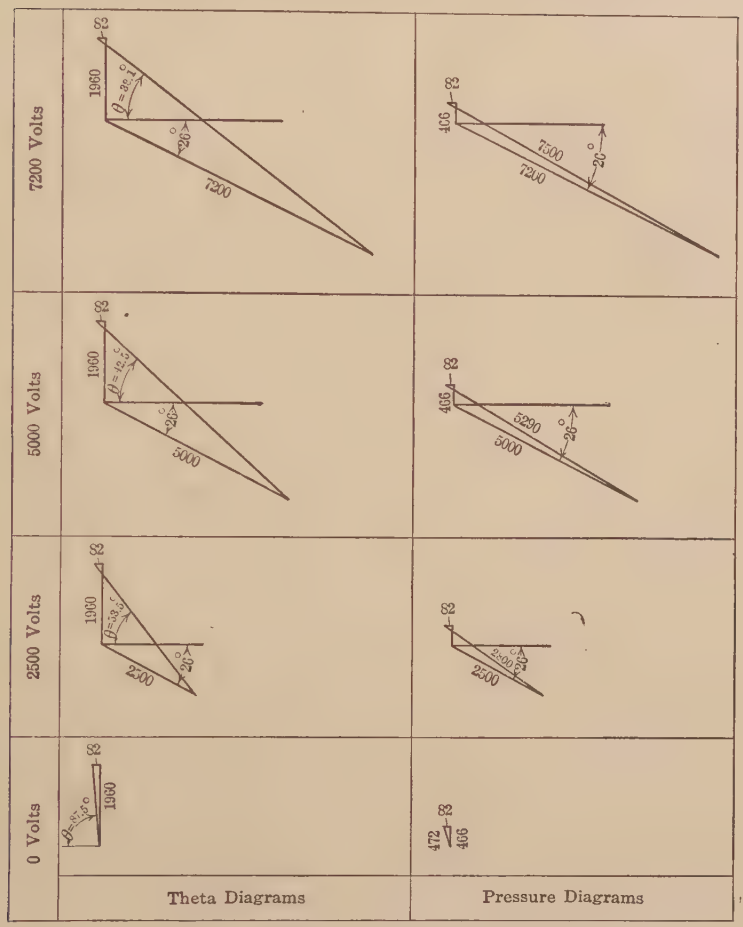


FIG. 43.—Theta and Pressure Diagrams for $I = 120$ and $G = 0.90$.

We are now in a position to obtain the total ats. for each value of the phase pressure. The steps are shown in the following table:

Phase Pressure.	Saturation Ats.	Ats. Required to Offset Armature Demagnetization.	Total Ats. for 120 Amp. and 0.90 Power-factor.
0	1 000	7200	8 200
2500	5 000	5780	10 780
5000	9 300	4850	14 150
6950	16 700	4450	21 150
7200	19 200	4440	23 640

These values for 120 amperes at 0.90 power-factor and those previously obtained for 120 amperes at unity power-factor, give us the two load-saturation curves plotted in Fig. 44. We see that for a phase pressure of 6950 volts, when the current is 120 amperes and at 0.90 power-factor, the required excitation is 21 150 ats. From the no-load saturation curve we find that an

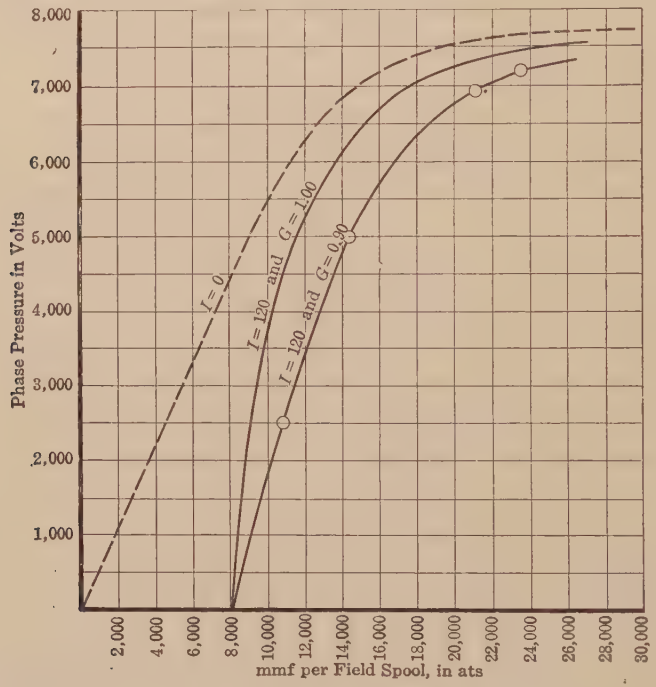


FIG. 44.—Saturation Curves for $I = 120$ and for $G = 1.00$ and 0.90.

excitation of 21 150 ats. occasions, at no load, a phase pressure of 7600 volts

$$\frac{7600 - 6950}{6950} \times 100 = 9.4.$$

The inherent regulation at 0.90 power-factor is, for this machine, 9.4 per cent. In other words, if, for an output of 120 amperes at a power-factor of 0.90 we adjust the excitation to such a value as to give a phase pressure of 6950 volts, and if, keeping

the excitation constant at this value, the load is decreased to zero, the pressure will rise 9.4 per cent.

ESTIMATION OF SATURATION CURVE FOR 120 AMPERES AT A POWER FACTOR OF 0.80

Now let us carry through precisely similar calculations for 120 amperes at a still lower power-factor, namely, a power-factor of 0.80. We shall first estimate the required mmf. (at 120 amperes and 0.80 power-factor) for phase pressures of 7200, 5000, 2500 and 0 volts, and from these four results we can plot the required saturation curve.

We have the relation:

$$\cos^{-1} 0.80 = 37.0°.$$

The reactance voltage and the internal IR drop remain the same as in the diagrams of Fig. 43. From these data we readily arrive at the diagrams of Fig. 45.

From the internal-pressure diagrams in Fig. 45 and from the no-load saturation curve in Fig. 23, we arrive at the following results:

Phase Pressure.	Internal Pressure.	Saturation Ats.
0	472	1 000
2500	2 850	5 100
5000	5 330	9 400
7200	7 560	20 000

From the θ diagrams in Fig. 45 we obtain the following results:

Phase Pressure.	θ.	Sin θ.	Ats Required to Offset Armature Demagnetization.
0	87.5°	0.999	7200
2500	60.2°	0.868	6250
5000	51.0°	0.777	5600
7290	47.5°	0.737	5300

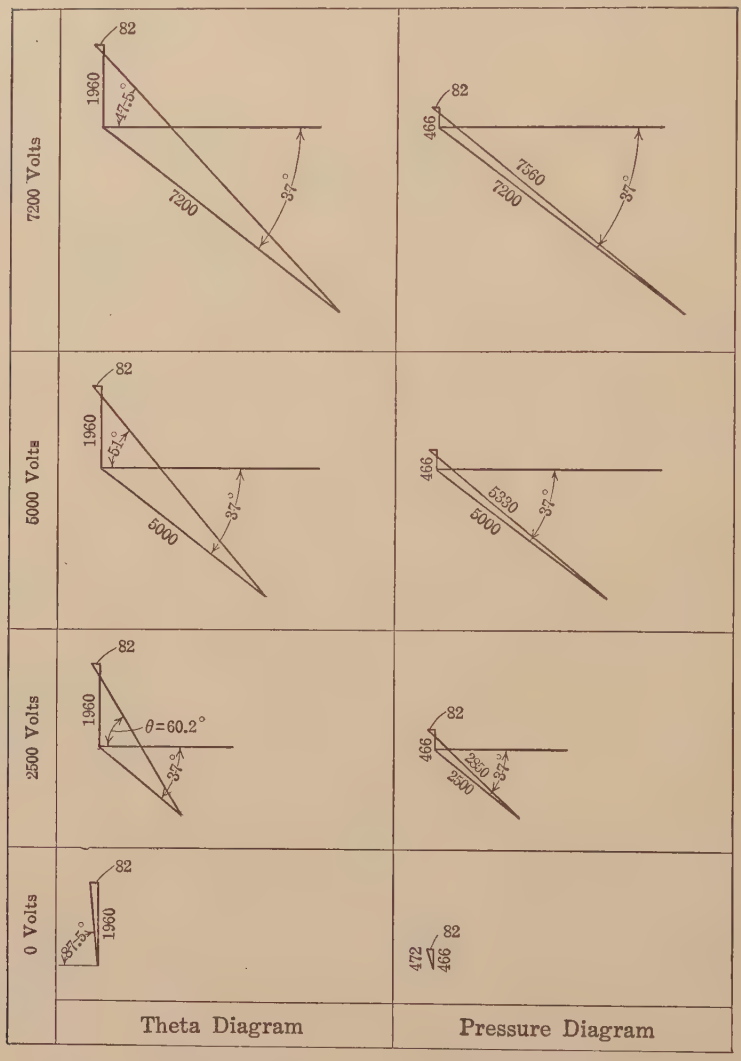


FIG. 45.—Theta and Pressure Diagrams for $I=120$ and $G=0.80$.

From the data in the two preceding tables, we can obtain the total ats. for each value of the phase pressure. This is worked through in the following table:

Phase Pressure.	Saturation Ats.	Ats. Required to Offset Armature Demagnetization.	Total Ats. for 120 Amperes and 0.80 Power-factor.
0	1 000	7200	8 200
2500	5 100	6250	11 350
5000	9 400	5600	15 000
7200	20 000	5300	25 300

CALCULATION OF SATURATION CURVE FOR ZERO POWER FACTOR AND 120 AMPERES

For 120 amperes at zero power-factor, the calculations are so simple that no diagrams need be drawn. The internal pressures are:

472 volts for a phase pressure of..... 0 volts
(466+2500=)2966 volts for a phase pressure of 2500 "
(466+5000=)5466 volts for a phase pressure of 5000 "
(466+7200=)7666 volts for a phase pressure of 7200 "

Theta is Equal to 90° for Loads of Zero Power-factor. The angle θ, by which the conductors have passed beyond mid-pole-face position when the instant arrives at which they are carrying the crest current, is substantially 90° for all four cases, and consequently the ats. required to offset armature demagnetization remain constant at 7200. The calculations are completed in the following table:

Phase Pressure.	Internal Pressure.	Saturation Ats.	Ats. Required to Offset Armature Demagnetization.	Total Ats. for 120 Amperes for Zero Power-factor.
0	472	1 000	7200	8 200
2500	2966	5 100	7200	12 300
5000	5466	9 900	7200	17 100
7200	7666	23 000	7200	30 200

In Fig. 46 are given saturation curves for loads of 120 amperes at 1.00, 0.90, 0.80 and 0 power-factors.

In Fig. 47 the same results are thrown into a set of curves, each relating to a particular phase-pressure, the ordinates representing excitation and the abscissae representing power-factor.

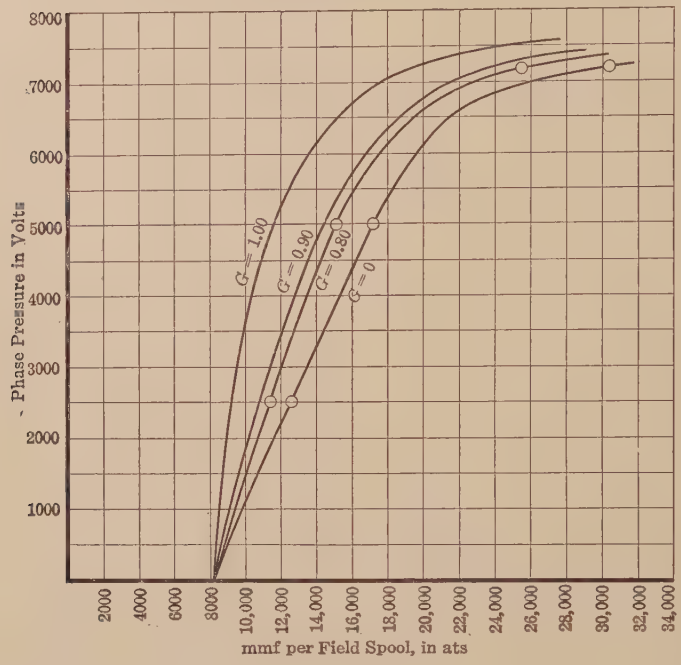


FIG. 46.—Saturation Curves for $I = 120$ at Various Power-factors.

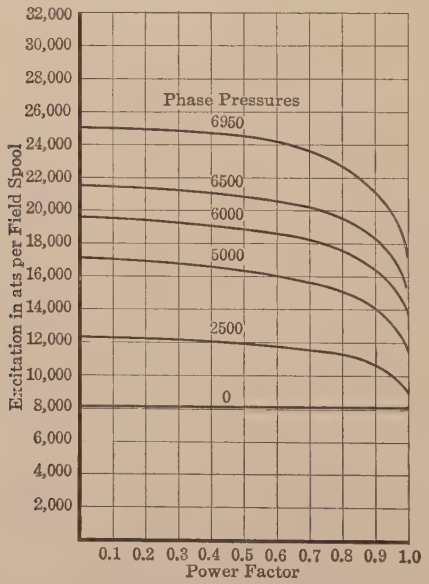


FIG. 47.—Curves showing the Required Excitation for $I = 120$ at Various Phase Pressures and Power-factors.

THE EXCITATION REGULATION

The inherent regulation is not the only kind of regulation which it is necessary to take into consideration in connection with the performance of a generator. There is also the "excitation regulation." This, for a given power-factor may be defined as the percentage increase in excitation which is required in order to maintain constant pressure when the output is increased from no-load to any particular specified value of the current.

For our design we have estimated that for a phase pressure of 6950 volts *at no load*, the required excitation is:

14 400 ats.

For this same phase pressure but with an output of 120 amperes per phase, the required excitations are:

17 140 ats. for $G=1.00$
21 150 ats. for $G=0.90$
22 600 ats. for $G=0.80$
25 000 ats. for $G=0$

The corresponding values of the excitation regulation are:

$\left(\frac{17\,140-14\,400}{14\,400}\times 100=\right)$ 19.0 per cent for $G=1.00$

$\left(\frac{21\,150-14\,400}{14\,400}\times 100=\right)$ 46.8 per cent for $G=0.90$

$\left(\frac{22\,600-14\,400}{14\,400}\times 100=\right)$ 57.0 per cent for $G=0.80$

$\left(\frac{25\,000-14\,400}{14\,400}\times 100=\right)$ 73.7 per cent for $G=0$

Excitation Regulation Curves. Curves plotted for given values of G, and of the phase pressure, with excitation as ordinates and with current output per phase as abscissae, are termed excitation regulation curves. We have values for such curves so far as relates to $I=0$ and $I=120$, but with respect to higher values, we have but one point, namely:

$I=240$
Phase pressure $=6950$
$G=1.00$
Excitation $=23\,000$.

Let us work out corresponding values for $I=240$ and with the other power-factors, namely,

$$G=0.90,\ G=0.80 \text{ and } G=0.$$

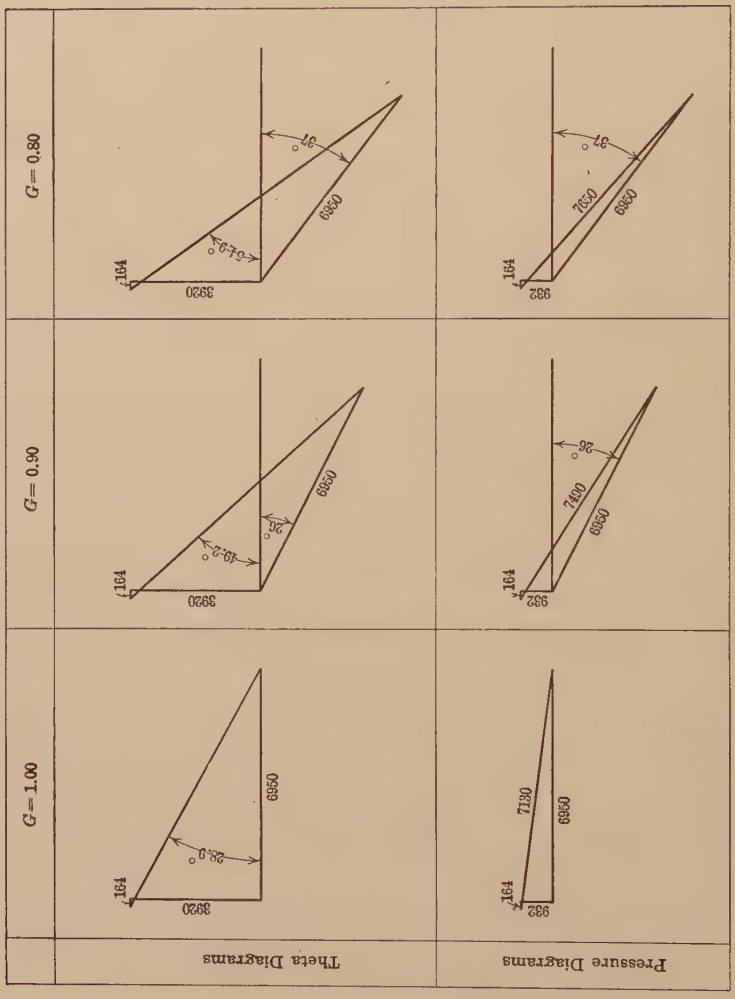


Fig. 48.—Theta and Pressure Diagrams for 240 Amperes and 6950 Volts.

For these power-factors and also for $G=1.00$, the theta and pressure diagrams are drawn in Fig. 48. With the values obtained from these diagrams the estimates may be completed as follows:

G.	θ	Sin θ.	Ats. for Offsetting Armature Demagnetization.	Internal Phase Pressure.	Saturation Ats.	Total Ats.
1.00	28.9°	0.483	7 000	7130	15 600	22 730
0.90	49.2°	0.758	10 900	7490	18 900	26 390
0.80	54.9°	0.817	11 800	7650	22 200	29 850
0	90°	1.00	14 400	7880	35 000	43 000

The values in the last column and corresponding values already obtained for $I=120$ and for $I=0$, are brought together in the following table:

G.	Excitation for 6950 Volts and:		
	$I=0$.	$I=120$.	$I=240$.
1.00	14 400 ats.	17 140 ats.	22 730 ats.
0.90	14 400 "	21 150 "	26 390 "
0.80	14 400 "	22 600 "	29 850 "
0	14 400 "	25 000 "	43 000 "

These values are plotted in the excitation regulation curves of Fig. 49.

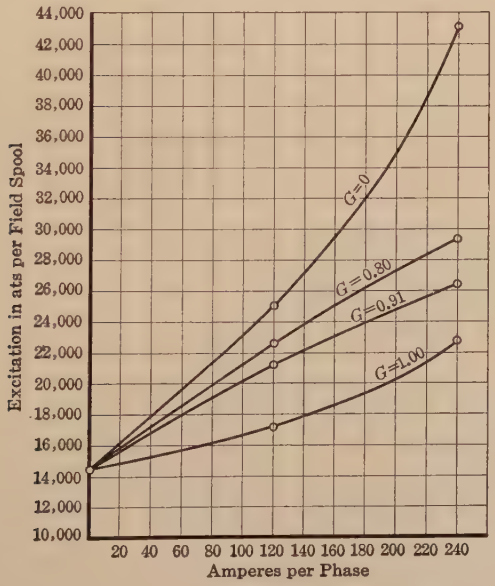


FIG. 49.—Excitation Regulation Curves for 6950 Volts.

Saturation Curves for 240 Amperes. In the course of the previous investigation we have had occasion to obtain the excitation required for a phase pressure of 6950 volts and with an output of 240 amperes. These values are:

G	Excitation.
1.00	22 730 ats.
0.90	26 390 "
0.80	29 850 "
0	43 000 "

It is not necessary to indicate the steps in working out corresponding values for 5000 and 2500 volts, and it will suffice to state simply that the values are those set forth below:

G.	Excitation for Phase Pressures of:			
	0 Volts.	2500 Volts.	5000 Volts.	6950 Volts.
1.00	16 300 ats.	17 000 ats.	18 200 ats.	22 730 ats.
0.90	16 300 "	19 000 "	21 700 "	26 390 "
0.80	16 300 "	19 500 "	22 700 "	29 850 "
0	16 300 "	20 600 "	25 400 "	43 000 "

These 240-ampere saturation curves are plotted in Fig. 50.

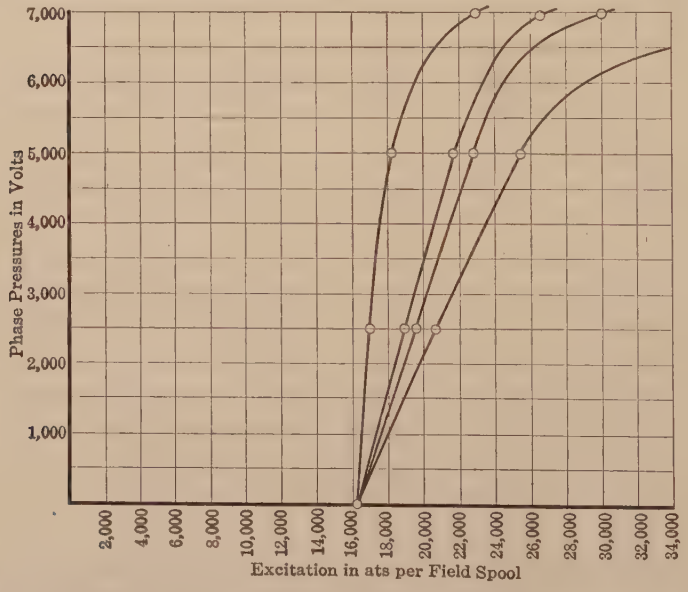


FIG. 50.—Saturation Curves for Various Values of the Power-factor and for $I = 240$.

VOLT-AMPERE CURVES

From the data in Figs. 23, 46 and 50, relating respectively to saturation curves for $I=0$, $I=120$ and $I=240$, we can construct curves which may be designated "volt-ampere"

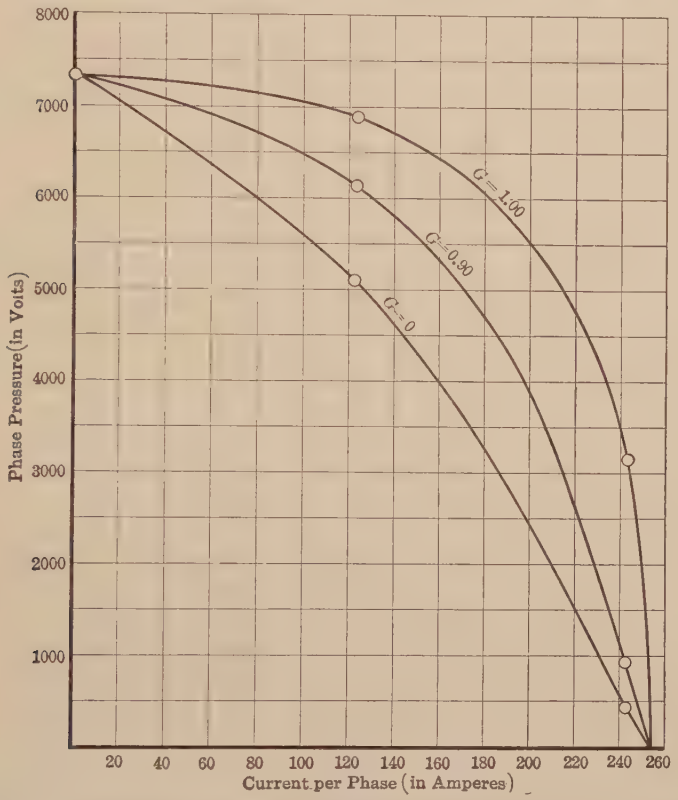


FIG. 51.—Volt-ampere Curves for Various Power-factors and for a mmf. of 17 140 ats. per Field Spool.

curves, since they are plotted with the phase pressure in volts as ordinates and with the current per phase, in amperes, as abscissae.

For any particular volt-ampere curve the excitation and the power-factor are constants. For the volt-ampere curves in Fig. 51, the excitation is maintained constant at 17 140 ats., the mmf. required at 6950 volts, 120 amperes and unity power-factor.

Comment is required on the matter of the value at which the volt-ampere curves cut the axis of abscissae. This is seen to be at 254 amperes. For this current, the mmf. required to overcome armature demagnetization is obviously:

$$\frac{254}{120}\times 7200 = 15\,300 \text{ ats.}$$

Since the excitation is maintained constant at 17 140 ats., there is a residue of:

$$17\,140 - 15\,300 = 1840 \text{ ats.},$$

and this suffices to provide the flux corresponding chiefly to the reactance of the end connections. We have seen on pp. 51 and 53 that the reactance of the end connections amounts to:

$$(16.3\times 0.238 =) 3.88 \text{ ohms.}$$

Consequently for 254 amperes the reactance voltage is:

$$254\times 3.88 = 990 \text{ volts.}$$

The IR drop is:

$$254\times 0.685 = 174 \text{ volts.}$$

The impedence voltage on short-circuit with 250 amperes is consequently:

$$\sqrt{990^2+174^2} = 1000 \text{ volts.}$$

From the no-load saturation curve of Fig. 23, we see that 1800 ats. are required for a phase pressure of 1000 volts.

THE SHORT-CIRCUIT CURVE

When the stator windings are closed on themselves with no external resistance, then the field excitation required to occasion a given current in the armature windings must exceed the armature mmf. by an amount sufficient to supply a flux corresponding to the impedance drop.

The impedance is made up of two parts:

THE REACTANCE OF THE END CONNECTIONS AND THE RESISTANCE OF THE WINDINGS

We have seen (on p. 51) that the reactance of one phase at 25 cycles, is 16.3 ohms. Furthermore we have seen (on p. 53) that the reactance of the end connections is 23.8 per cent of this value, or:

$$0.238 \times 16.3 = 3.88 \text{ ohms.}$$

Also we have seen (on p. 22) that the resistance per phase, at 60° C., is:

$$0.665 \text{ ohm.}$$

Consequently, at 25 cycles, the impedance is:

$$\sqrt{3.88^2 + 0.665^2} = 3.94 \text{ ohms.}$$

For any particular value of the current, the impedance drop is obtained by multiplying the current by 3.94 ohms. Thus for 100 amperes we have an impedance drop of:

$$100 \times 3.94 = 394 \text{ volts.}$$

From the no-load saturation curve of Fig. 23, we find that for a pressure of 394 volts per phase, a mmf. of 700 ats. per field spool is required.

There are 25 turns per pole per phase. Consequently for a current of 100 amperes per phase, the armature mmf. amounts to:

$$2.4 \times 25 \times 100 = 6000 \text{ ats.}$$

Thus to send 100 amperes per phase through the short-circuited stator windings, there is required a mmf. of:

$$700 + 6000 = 6700 \text{ ats. per field spool.}$$

Making corresponding calculations for 200 and 300 amperes we arrive at the following results, which are plotted in Fig. 52:

Current in Armature.	Impedance Drop.	Saturation mmf.	Armature mmf.	Total Required Ats.
0 amp.	0 volts	0 ats.	0 ats.	0 ats.
100 "	394 "	700 "	6 000 "	6 700 "
200 "	788 "	1400 "	12 000 "	13 400 "
300 "	1182 "	2100 "	18 000 "	20 100 "

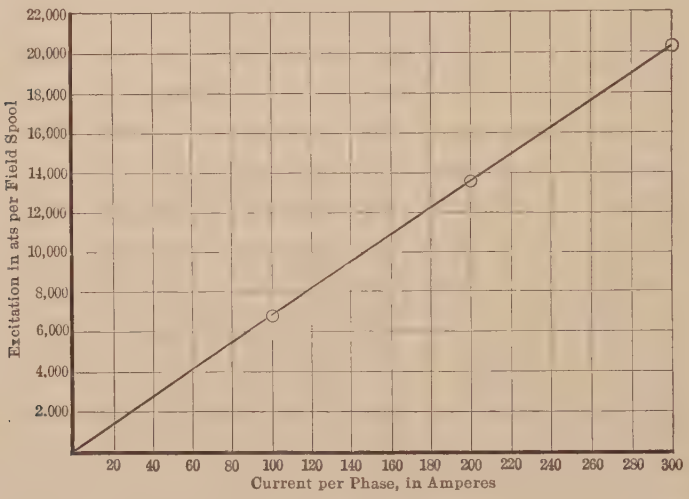


FIG. 52.—Short-circuit Curve for 2500-kva. Alternator with an 18-mm. Air-gap.

Up to 300 amperes, the short-circuit characteristic is a straight line, but at some exceedingly-high current values it may, in machines designed with strong fields, bend upward quite a little owing to saturation.

Let us now investigate the effect of employing a lower mmf. in our design.

INFLUENCE OF MODIFICATIONS IN THE NO-LOAD SATURATION CURVE

Let us make the single change of decreasing the radial depth of the air-gap to 6 mm. in place of the original value of 18 mm. The component and resultant mmf. at no load will now differ from

those given in the table on p. 42 to the extent indicated in the following table:

	6000 Volts.	6950 Volts.	7500 Volts.
Air-gap	3340 ats.	3850 ats.	4180 ats.
Teeth	90 "	330 "	720 "
Magnet core	770 "	2000 "	5300 "
Yoke	200 "	320 "	420 "
Stator core	140 "	190 "	230 "
Total mmf	4540 ats.	6690 ats.	10 850 ats.

These values are plotted in the no-load saturation curve of Fig. 53.

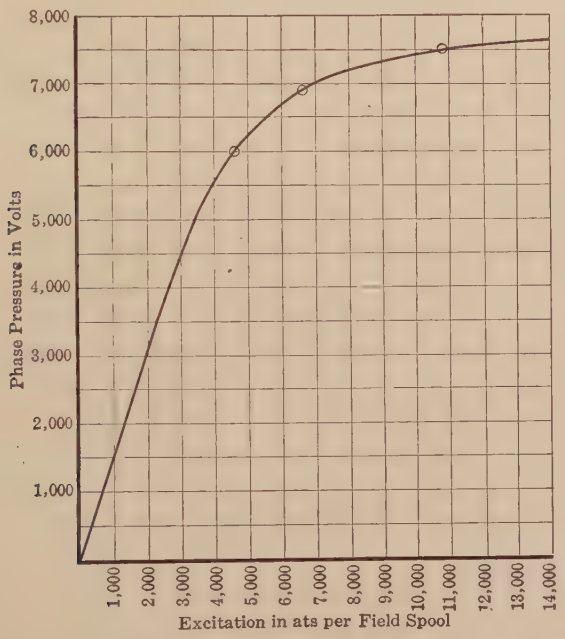


FIG. 53.—No-load Saturation Curve of 2500-kva. Alternator with a 6-mm. Air-gap.

The calculations for the load saturation curves at various power-factors are given in the table on p. 80.

	Phase Pressure.	Internal Pressure.	Saturation Ats.	Ats. for Offsetting Armature Demagnetization.	Total Required Ats.
$G=1.00$ $I=120$ $\Delta=6$	0	476	350	7200	7 550
	2500	2630	1 700	4350	6 050
	5000	5100	3 580	2950	6 530
	6950	7050	7 200	1940	9 140
	7500	7590	12 900	1800	14 700
$G=0.90$ $I=120$ $\Delta=6$	0	476	350	7200	7 550
	2500	2800	1 800	5780	7 580
	5000	5290	3 700	4850	8 550
	6950	7250	8 300	4450	12 750
	7200	7500	10 850	4440	15 290
$G=0.80$ $I=120$ $\Delta=6$	0	476	350	7200	7 550
	2500	2850	1 820	6250	8 070
	5000	5330	3 800	5600	9 400
	7200	7560	12 000	5300	17 300
$G=0$ $I=120$ $\Delta=6$	0	476	350	7200	7 550
	2500	2966	1 900	7200	9 100
	5000	5466	3 900	7200	11 100
	7200	7666	15 500	7200	22 700

All these saturation curves are brought together in the curves in Fig. 54.

From Figs. 46 and 54 we obtain the following data:

G.	Mmf. per Field Spool Required for 120 Amp. and a Phase Pressure of 6950 Volts.	
	For $\Delta=18$.	For $\Delta=6$.
1.00	17 100	9 140
0.90	21 000	12 750
0.80	22 600	14 400
0	25 000	17 800

At first thought it would appear to be an advantage to obtain the required full-load conditions with the much smaller amount of field copper which would suffice with the 6-mm. air-gap. Furthermore the inherent regulation at full load is identical with that for the 18-mm. air-gap. Let us test this last assertion by obtaining from Figs. 23 and 53 the no-load pressures for the

eight values of the excitation given in the preceding table. We have:

G.	No Load Pressures Corresponding to the Excitations Required for 6950 Volts and 120 Amperes.	
	$\Delta = 18$.	$\Delta = 6$.
1.00	7350	7350
0.90	7600	7600
0.80	7650	7650
0	7700	7700

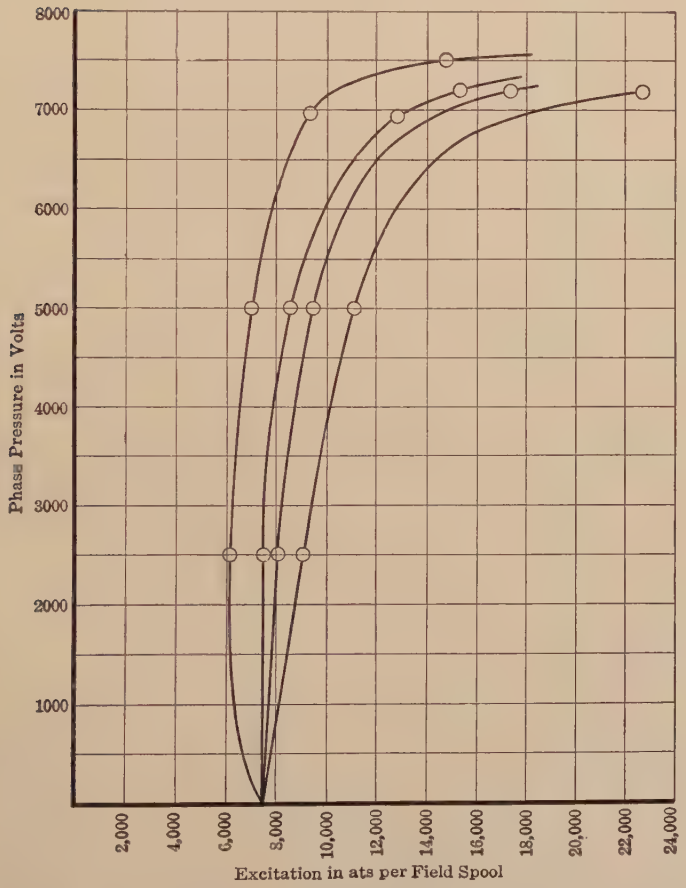


FIG. 54.—Load Saturation Curves for 2500-kva. Alternator with 6-mm. Air-gap.

This is a consequence of the plan of limiting the pressure rise by the saturation of the magnetic circuit. The inherent regulation for both air-gaps, has, for full-load current of 120 amperes, the values given in the following table:

$G=1.00$ Inherent regulation $=\frac{7350-6950}{6950}\times 100=$ 5.8 per cent.

$G=0.90$ " " $=\frac{7600-6950}{6950}\times 100=$ 9.4 "

$G=0.80$ " " $=\frac{7650-6950}{6950}\times 100=10.1$ "

$G=0$ " " $=\frac{7700-6950}{6950}\times 100=10.8$ "

If there are objections to employing the smaller air-gap, we must look elsewhere for them. Let us examine for instance, into the question of the amount of current which, with normal excitation of 9140 ats. (corresponding to 6950 volts and 120 amperes at unity power-factor) could flow on short-circuit. Without going into the matter of the precise determination of the saturation mmf., let us assign to this quantity the reasonable value of 700 ats. This leaves:

$$9140-700=8440 \text{ ats.}$$

for offsetting armature demagnetization. We have seen that the resultant armature mmf. is 2.4 times the mmf. per pole per phase.

Thus we have:

$$\text{mmf. per pole per phase}=\frac{8440}{2.4}=3500 \text{ ats.}$$

Since we have 25 turns per pole per phase, the short-circuit current is only:

$$\frac{3500}{25}=140 \text{ amperes.}$$

Thus with an excitation of 9140 ats. (the value corresponding to full load at unity power-factor), a current overload of only:

$$\left(\frac{140-120}{120}\times 100=\right)16.7 \text{ per cent}$$

suffices to pull the terminal pressure down to 0 volts.

Similarly with an excitation of 14 400 ats., (the value for 120 amperes at 6950 volts and 0.80 power-factor with a 6-mm. air-gap), the short-circuit current is of the order of:

$$\frac{14\,400-1000}{2.4\times 25}=224 \text{ amperes.}$$

Even with this greater excitation, the current on short-circuit is less than twice full-load current.

While the modern tendency is toward designing with limited overload capacity, it is nevertheless impracticable to employ

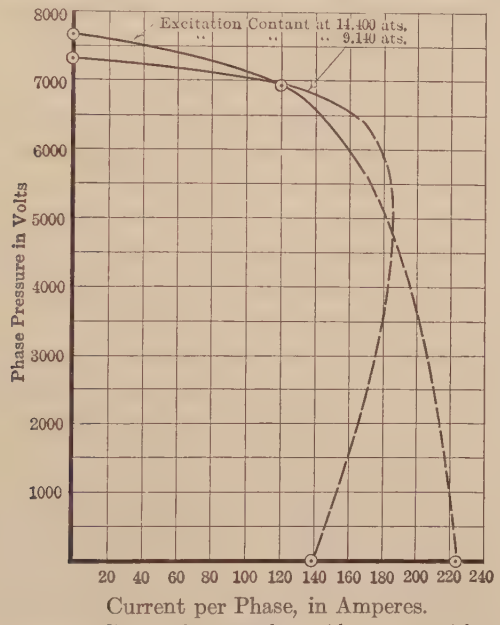


FIG. 55.—Volt-ampere Curves for 2500-kva. Alternator with 6-mm. Air-gap.

generators whose volt-ampere characteristics turn down nearly so abruptly as do those in Fig. 55 which represent these two values. In Fig. 55, the right-hand portions of the curves have been drawn dotted, as it has not been deemed worth while to carry through the calculations necessary for their precise predetermination.

Let us now revert to our original design with the 18-mm. air-gap which we have shown to possess the more appropriate attributes.

THE DESIGN OF THE FIELD SPOOLS

Our generator's normal rating is 2500 kva. at a power-factor of 0.90 and a phase pressure of 6950 volts. The current per phase is then 120 amperes. For these conditions the required excitation is:

21 150 ats. per field spool.

The field spools must be so designed as to provide this mmf. with an ultimate temperature rise of preferably not more than 45° Cent. above the temperature of the surrounding air.

The question of the preferable pressure to employ for exciting the field, is one which can only be decided by a careful consideration of the conditions in each case. The pressure employed in the electricity supply station for lighting and other miscellaneous purposes, is usually appropriate, although it is by no means out of the question that it may be good policy in some cases to provide special generators to serve exclusively as exciters. These exciters should, however, be independently driven. In other words, their speed should be independent of the speed of the generator for which the excitation is provided. It is the worst conceivable arrangement to have the exciter driven from the shaft of the alternator, as any change in the speed will then be accompanied by a more than proportional change in the excitation.

In general, the larger the generator or the more poles it has, the higher is the appropriate exciting pressure. But it is difficult to make any statement of this kind to which there will not be many exceptions.

Let us plan to excite our 2500-kva. generator from a 500-volt circuit and let us so arrange that when the machine is at its ultimate temperature of (20+45=) 65° Cent., 450 volts at the slip rings shall correspond to an excitation of 21 150 ats. The remaining (500−450=) 50 volts will be absorbed in the controlling rheostat. It would not be prudent to plan to use up the entire available pressure of 500 volts when obtaining the mmf. of 21 150 ats., for this would leave no margin for discrepancies between our estimates and the results which we should actually obtain on the completed machine.

Thus we have 450 volts for the eight spools in series, or:

$$\frac{450}{8} = 56.3 \text{ volts per spool.}$$

In Fig. 56 is shown a section through the magnet core and the spool.

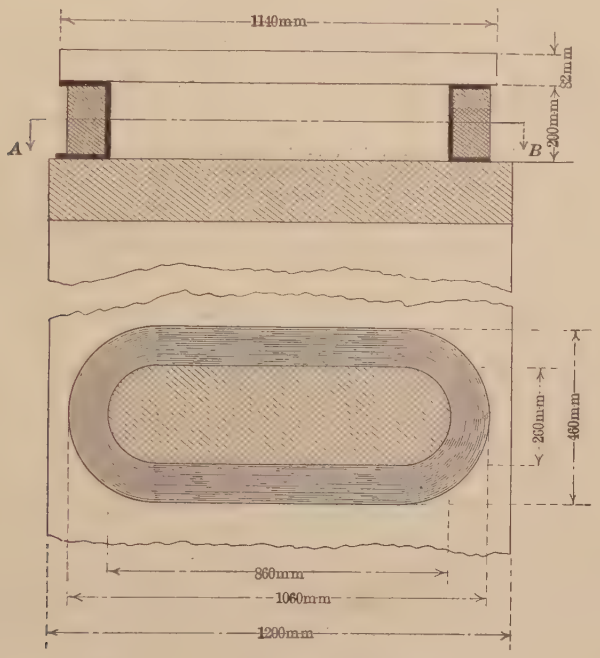


FIG. 56.—Sketches of Magnet Pole and Field Spool for 2500-kva. Alternator.

The inner periphery of the spool is

$$26 \times \pi + 2 \times 60 = 82 + 120 = 202 \text{ cm.}$$

The outer periphery is $46 \times \pi + 2 \times 60 = 144 + 120 = 264$ cm.
For the mean length of turn we have:

$$\text{mlt.} = \frac{202 + 264}{2} = 233 \text{ cm.}$$

Suppose we were to provide our normal excitation of 21 150 ats. by means of a single turn carrying 21 150 amperes. For this excitation the pressure per spool is 56.3 volts. Consequently the pressure at the terminals of our hypothetical turn carrying 21 150 amperes is also the 56.3 volts allocated to each of the eight spools. The resistance of the turn must consequently be:

$$\frac{56.3}{21\,150} = 0.00266 \text{ ohm.}$$

We have mlt. = 233 cm.

Therefore since the specific resistance of copper per centimeter cube, at 65° Cent., is 0.00000204, we have:

$$\text{Cross-section of the conductor} = \frac{233 \times 0.00000204}{0.00266}$$

$$= 0.179 \text{ sq.cm.}$$

Now if we were to provide the entire excitation by a single turn per spool as above suggested, the loss in field excitation would be:

$$\frac{500 \times 21\,150}{1000} = 10\,600 \text{ kw.}$$

This is over five times the output of our machine. Consequently its efficiency would be low—say some 18 per cent. But also, we should be running our conductor at a density of:

$$\frac{21\,150}{0.179} = 118\,000 \text{ amperes per sq.cm.}$$

and it would fuse long before this density could be reached. It would, in fact, fuse with a current of the order of only some 1000 amperes. Also the loss of 10 600 kw. in the field spools would suffice, even if the heat could be uniformly distributed through the whole mass of the machine, to raise it to an exceedingly high temperature. Even a very spacious engine room would be unendurable with so great a dissipation of energy taking place within it.

So let us look into the merits of employing 10 turns per spool instead of only one turn per spool.

Since we require an excitation of 21 150 ats. per spool, the current will, in this case, be only $\frac{21\,150}{10} = 2115$ amperes.

Since the mlt. is equal to 233 cm., the 10 turns will have a length of

$$10 \times 233 = 2330 \text{ cm.}$$

The resistance of the 10 turns must now be:

$$\frac{56.3}{2115} = 0.0266 \text{ ohm.}$$

Then we have:

$$\text{Section} = \frac{2330 \times 0.00000204}{0.0266} = 0.179 \text{ sq.cm.}$$

This is the same value as before. In fact, for, firstly, a given pressure at the terminals of a spool; secondly, a given excitation to be provided, and thirdly, a given mlt., the cross-section of the conductor is independent of the number of turns employed to provide that excitation, and it is convenient to determine upon the cross-section by first assuming that a single turn will be employed. Obviously the greater the number of turns per spool, the less will be the current, and since the terminal pressure is fixed, the less also will be the power. Thus with our second assumption of a 10-turn coil, and only 2115 amperes in the exciting circuit, the excitation loss is reduced to:

$$\frac{500 \times 2115}{1000} = 1060 \text{ kw.}$$

and the efficiency of our 2500-kva. machine would rise to over 65 per cent.

With the endeavor to obtain a reasonably low excitation loss, it is obviously desirable to employ as many turns as we can arrange in the space at our disposal. We have already seen that this space provides a cross-section of $10 \times 20 = 200$ sq.cm. Our conductor has a cross-section of 17.9 sq. mm. and should thus

have a bare diameter of 4.77 mm. The curves in Fig. 57 give the thicknesses of the insulation on single, double and triple cotton-covered wires of various diameters.

If, in this case, we employ a double cotton covering, the insulated diameter will be $4.77+(2\times0.18)=5.13$ mm. The term

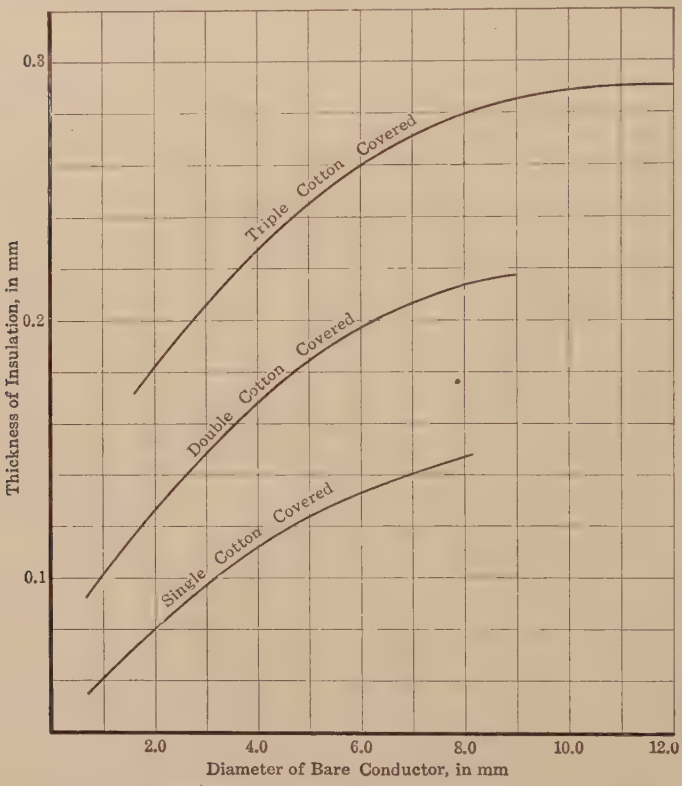


Fig. 57.—Thicknesses of Insulation on Cotton-covered Wires.

"space factor" as applied to field spools, is employed to denote the ratio of the total cross-section of copper in the spool, to the gross area of cross-section of the winding space. Attainable values for the "space factor" of spools wound with wires of various sizes and with various insulations, are given in the curves in Fig. 58.

For the case we are considering, we ascertain from the curves that the space-factor may be equal to 0.55. That is to say: 55

per cent of the cross-section of the winding space will be copper and the remaining 45 per cent will be made up of the insulation and the waste space. Thus the aggregate cross-section of copper will be $200 \times 0.55 = 110$ sq.cm. Consequently the number of turns is:

$$\frac{110}{0.179} = 615.$$

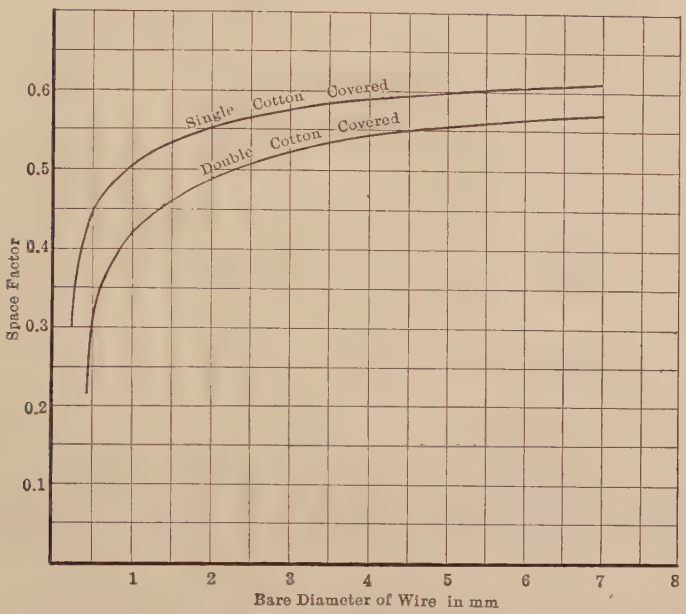


FIG. 58.—Curves showing "Space Factors" of Field Spools Wound with Wires of Various Diameters.

For the normal magnetomotive force of 21 150 ats. per spool, the exciting current is:

$$\frac{21\,150}{615} = 34.4 \text{ amperes.}$$

The total excitation loss is:

$$500 \times 34.4 = 17\,200 \text{ watts.}$$

But of this loss of 17 200 watts:

$$\frac{50}{500}\times 17\ 200 = 1720 \text{ watts}$$

are dissipated in the field regulating rheostat, and only:

$$\frac{450}{500}\times 17\ 200 = 15\ 500 \text{ watts}$$

are dissipated in the field spools. The loss per spool is thus:

$$\frac{15\ 500}{8} = 1940 \text{ watts.}$$

The next step is to ascertain whether this will consist with a suitably-low temperature rise. The peripheral speed of our rotating field is:

$$1.78\times\pi\times\frac{375}{60} = 35.0 \text{ meters per second.}$$

At this speed and with this very open general construction with salient poles, it will be practicable to restrict the temperature rise to some 1.3° rise per watt per sq.dm. of external cylindrical radiating surface of the field spool. We have:

External periphery of the spool = 26.4 dm.

Length of spool = 2.0 dm.

External cylindrical radiating surface = $26.4\times 2.0 = 52.8$ sq.dm.

$$\text{Watts per sq.dm.} = \frac{1940}{52.8} = 36.8.$$

Ultimate temperature rise = $36.8\times 1.3 = 48°$ Cent.

This is a high value and would not be in accordance with the terms of usual specifications. But the modern tendency is to take advantage of the increasing knowledge of the properties of insulating materials and to permit higher temperatures in low-pressure windings, provided offsetting advantages are thereby

obtained. This is the case in moderate-speed and high-speed polyphase generators. The entire design profits in great measure by compressing the rotor into the smallest reasonable compass. In designing on such lines, the space available for the field spools is necessarily restricted. In the case of the design under consideration, we could, by decreasing the air-gap, decrease the required excitation and consequently also the temperature rise of the field spools. But we have already seen that the characteristics of the machine would be impaired by doing this.

For a power-factor of 1.00 the mmf. required per field spool is only 17 200 ats. The loss in the field spool decreases as the square of the mmf. Consequently were the machine required for an output of 2500 kva. at *exclusively unity power-factor*, the temperature rise would be only:

$$\left(\frac{17\,200}{21\,150}\right)^2 \times 48° = 32° \text{ Cent.}$$

Thus for an putput of 2500 **kilowatts** at unity power-factor, the field spools are actually 33 per cent cooler than for an output of only 2250 **kilowatts** at a power-factor of 0.90 (i.e., for an output of 2500 kva. and a power-factor of 0.90).

If on the contrary, the machine were required for an output of only 2000 **kilowatts** but at a power-factor of 0.80 (again 2500 kilovolt-amperes), the temperature rise of the field spools would be:

$$\left(\frac{22\,500}{21\,150}\right)^2 \times 48 = 54° \text{ Cent.}$$

It is impressive to note that although this output (2000 **kilowatts** at 0.80 power-factor), is 20 per cent less than an output of 2500 **kilowatts** at unity power-factor, the temperature rise of the field spools is:

$$\frac{54-32}{32} \times 100 = 69 \text{ per cent}$$

greater. The losses and temperature rise in the other parts of the machine will be the same for both these conditions, since the **kilovolt-ampere** output of the machine is 2500, in both cases.

It is the object of this treatise to explain methods of design, and consequently no particular advantage would be gained by carrying through modified calculations for the purpose of providing field spools which would permit of carrying the rated load at 0.90 power-factor with a temperature rise of only 45° Cent. instead of a temperature rise of 48° Cent. An inspection of Figs. 21 and 56 shows that there is room for more spool copper should its use appear desirable. It would also be practicable to decrease the internal diameter of the magnet yoke from the present 928 mm. down to—say—878 mm. and increase the radial length of the magnet core (and consequently also the length of the winding space), by 25 mm.

THE CORE LOSS

In polyphase generators, the core losses may be roughly predetermined from the data given in Table 10.

TABLE 10. DATA FOR ESTIMATING THE CORE LOSS IN POLYPHASE GENERATORS.

Density in Stator Core in Lines per Square Centimeter.	Core Loss in Stator Core, in kw. per (Metric) Ton for Various Periodicities.		
	$\sim$ =15.	$\sim$ =25.	$\sim$ =50.
6 000	1.1	2.2	5.0
8 000	1.7	3.0	7.4
10 000	2.2	4.0	10.0
12 000	2.6	5.2	
14 000	3.2	6.2	

In the case of the 2500-kva., 25-cycle generator which we have taken for our example, the density in the stator core is 10 000, lines per square centimeter. The core loss will thus be on the basis of some 4.0 kw. per (metric) ton. We must now estimate the weight of the stator core:

Estimation of Weight of Stator Core.

External diameter of stator core = 230 cm.
Internal diameter of stator core = 178 cm.

Area of the surface of an annular ring with the above external and internal diameters is equal to:

$$\frac{\pi}{4}(230^2-178^2)=16\,700 \text{ sq.cm.}$$

Area of the 120 slots $=120\times5.72\times2.41=1650$ sq.cm.

Net area of surface of stator core plate $=16\,700-1650$

$=15\,050$ sq.cm.

$$\lambda n=79.2 \text{ cm.}$$

$$\text{Volume of sheet steel in stator core}=\frac{79.2\times15\,050}{1\,000\,000}=1.19 \text{ cu.m.}$$

Weight of 1 cu.m. of sheet iron $=7.8$ (metric) tons.

Weight of stator core $=7.8\times1.19=9.3$ (metric) tons.

On the basis of a loss of 4.0 kw. per ton, we have:

$$\text{Core loss}=4.0\times9.3=37.2 \text{ kw.}$$

Friction Loss. No simple rules can be given for estimating the windage and bearing friction loss. The ability to form some rough idea of the former can, in a design of this type, only be acquired by long experience. It must suffice to state that, for the present design, a reasonable value is:

Windage and bearing friction loss $=20$ kw.

Excitation Loss. The loss in the exciting circuit is made up (see p. 90) of:

Loss in regulating rheostat $=1.7$ kw.

Loss in field spools $=15.5$ kw.

Stator I^2R Loss. The resistance of the stator winding (see p. 22) is:

0.665 ohm per phase.

and consequently we have:

$$\text{Stator } I^2R \text{ loss at rated load} = \frac{3 \times 120^2 \times 0.665}{1000} = 28.6 \text{ kw.}$$

Total Loss. The total loss at full load is the sum of these various losses:

I.	Stator I^2R	=	28 600 watts
II.	Field spool I^2R	=	15 500 "
III.	Field rheostat I^2R	=	1 700 "
IV.	Core loss	=	37 200 "
V.	Friction loss	=	20 000 "
	Total loss at full load	=	103 000 "
	Output at full load	=	2 250 000 "
	Input at full load	=	2 353 000 "

$$\text{Full-load efficiency} = \frac{2250}{2353} \times 100 = 95.6 \text{ per cent.}$$

CONSTANT AND VARIABLE LOSSES

Of the five component losses, the last four remain fairly constant at all loads, whereas the first loss (the stator I^2R loss) varies as the square of the load. It is true that the sum of the second and third losses decreases slightly with decreasing load, but in the present machine the total decrease is only in the ratio of the mmf. at full load and no load. The two mmf. are:

21 150 ats. at rated load.
14 400 ats. at no load.

The corresponding values of the total excitation loss are:

17 200 watts at full load (and 6950 volts per phase)

and

$\left(\frac{14\,400}{21\,150} \times 17\,200 =\right)$ 11 700 watts at no load (and 6950 volts per phase)

Thus the sum of the last four losses decreases from:

$$17\,200+37\,200+20\,000=74\,400 \text{ watts at full load}$$

down to

$$11\,700+37\,200+20\,000=68\,900 \text{ watts at no load.}$$

This decrease only amounts to

$$\frac{74\,400-68\,900}{74\,400}\times 100=7.4 \text{ per cent.}$$

Thus, taken broadly, we may take the last four component losses as making up an aggregate which we may term the "constant loss," and in contradistinction we may term the first component the "variable loss." In our design we have:

$$\text{Variable loss} = 28\,600 \text{ watts.}$$

$$\text{Constant loss} = 74\,400 \text{ watts.}$$

If we ignore the 7 per cent decrease in the constant loss, we may readily obtain the efficiencies at various loads. The method will be clear from an inspection of the following estimates:

EFFICIENCY AT ONE-FOURTH OF FULL LOAD

Variable loss $=0.25^2\times 28\,600$	$= 1\,800$ watts
Constant loss	$= 74\,400$ "
Total loss at one-fourth of full load	$= 76\,200$ "
Output $=0.25\times 2\,250\,000$	$=564\,000$ "
Input	$=640\,200$ watts

$$\eta\text{(at one-fourth load)} = \frac{564\,000}{640\,200}=0.882.$$

Efficiency at Half Load

Variable loss $= 0.50^2 \times 28\,600 =$	7 150	watts
Constant loss	$=$ 74 400	"
Total loss at half load	$=$ 81 550	"
Output at half load	$=1\,125\,000$	"
Input	$=1\,207\,000$	watts
η(at half load)	$=\frac{1125}{1207}=0.931.$	

Efficiency at 50 Per Cent Overload

Variable loss $= 1.50^2 \times 28\,600$	$=$ 64 500	watts
Constant loss	$=$ 74 400	"
Total loss at 50 per cent overload	$=$ 138 900	"
Output at 50 per cent overload	$=3\,380\,000$	"
Input	$=3\,519\,000$	watts
η(at 50% overload)	$=\frac{3380}{3519}=0.961.$	

Load Corresponding to Maximum Efficiency

When the variable losses have increased until they equal the constant losses, the efficiency will be at its maximum. The corresponding load is:

$$\sqrt{\frac{74\,400}{28\,600}} \times 2250 = 1.61 \times 2250$$

$$= 3640 \text{ kw.}$$

The efficiency is then:

$$\frac{3640}{3640+74.4+74.4} = \frac{3640}{3789} = 0.962.$$

From this point upward, the efficiency will decrease.

This brief method of estimating the efficiencies at several loads, gives slightly too low results at low loads and slightly too high results at high loads. But the errors are too slight to be of practical importance; in fact the inevitable errors in determining the component losses are of much greater magnitude.

In Fig. 59 is plotted an efficiency curve for the above-calculated values which correspond to a power-factor of 0.90.

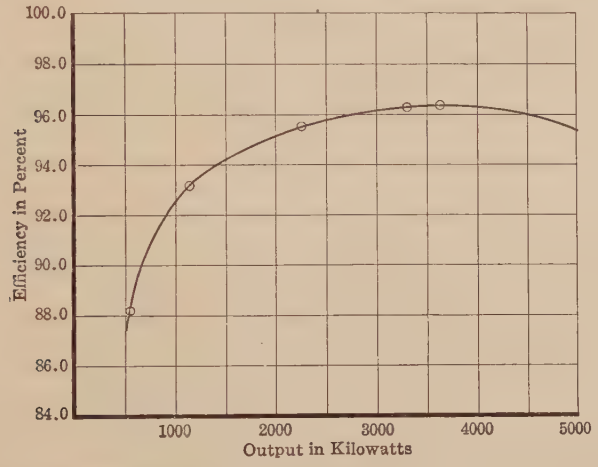


FIG. 59.—Efficiency Curve of 2500-kva. Alternator for $G = 0\ 90$.

DEPENDENCE OF EFFICIENCY ON POWER-FACTOR OF LOAD

Let us consider the load to be maintained at 2500 kva. but with different power-factors. The field excitation will be:

For $G = 1.00$:

$$\frac{17\ 200}{21\ 150} \times 17\ 200 = 14\ 000 \text{ watts.}$$

For $G = 0.90$:

$$17\ 200 \text{ watts.}$$

For $G = 0.80$:

$$\frac{22\ 500}{21\ 150} \times 17\ 200 = 18\ 300 \text{ watts.}$$

The total losses become:

G	Total Losses
1.00	99 800 watts
0.90	103 000 "
0.80	104 100 "

The outputs, inputs and efficiencies become:

G.	Output Corresponding to 2500 kva.	Input.	η
1.00	2500 kw.	2600 kw.	0.962
0.90	2250 "	2353 "	0.956
0.80	2000 "	2104 "	0.949

This brings out the importance of employing a considerable proportion of over-excited synchronous motors to offset the lagging load corresponding to induction motors. This question is further discussed in Chapter VI, entitled "Synchronous Motors *versus* Inductor Motors."

CHAPTER III

POLYPHASE GENERATORS WITH DISTRIBUTED FIELD WINDINGS

The type of polyphase generator with salient poles which has been described in the last chapter has served excellently as a basis for carrying through a set of typical calculations. Salient-pole generators are chiefly employed for slow- and medium-speed ratings. But for the high speeds associated with steam-turbine-driven sets, rotors with distributed field windings are practically universally employed in modern designs. It is not proposed to carry through the calculations for a design of this type. While there are a good many differences in detail, to which the professional designer gives careful attention, the underlying considerations are quite of the same nature as in the case of salient-pole designs. In Figs. 60 and 61 are shown photographs of a salient-pole rotor and a rotor with a distributed field winding. The former (Fig. 60), is for a medium speed (514 r.p.m.), water-wheel generator with a rated capacity for 1250 kva. The latter (Fig. 61), is for a 750-r.p.m., steam-turbine-driven set, with a rated capacity for 15000 kva. The former has 14 poles and the latter 4 poles. The former is for a periodicity of 60 cycles per second, and the latter for a periodicy of 25 cycles per second.

An inherent characteristic of high-speed sets relates to the great percentage which the sum of the core loss, windage and excitation bears to the total loss. In our 2500-kva. salient-pole design for 375 r.p.m., the core loss amounted to some 37 000 watts, the windage and bearing friction to 20 000 watts, and the excitation to 17 000 watts, making an aggregate of 74 000 watts for the "constant" losses, out of a total loss at full-load of 103 000 watts. But in a design for 2500 kva. at 3600 r.p.m., (i.e., for very nearly 10 times as great a speed), the core loss,

bearing-friction, windage, and excitation would together amount to some 72 000 watts out of a total of some 80 000 watts. A

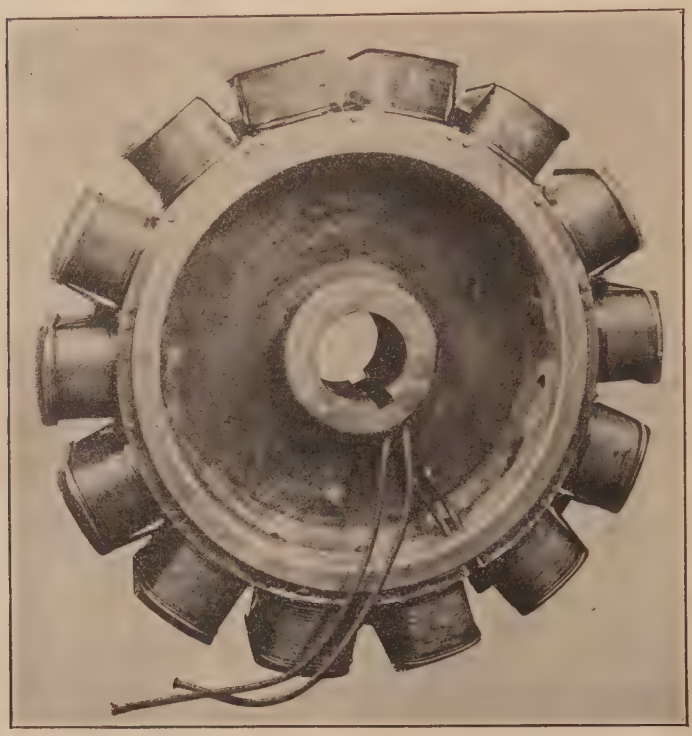


FIG. 60.—Salient Pole Rotor for a 14-pole, 1250-kva., 60-cycle, 514 r.p.m., 3-phase Alternator, built by the General Electric Co. of America.

representative distribution of the losses for a 2500-kva., 0.90-power-factor, 12 000-volt polyphase generator would be:

Armature I^2R loss	8 000	watts
Excitation I^2R loss	9 000	"
Core loss	32 000	"
Windage and bearing friction loss	31 000	"
Total loss at full load	80 000	"
Output at full load	2 250 000	"
Input at full load	2 330 000	"
Efficiency at full load	96.6	per cent

A result of the necessarily large percentage which the "constant" losses bear to the total losses, is that the efficiency falls off badly with decreasing load. In this instance we have:

Variable losses = 8 kw.

Constant losses = 72 kw.

The efficiencies at various loads work out as follows:

Load.	Efficiency.
$\frac{1}{4}$	88.5 per cent
$\frac{1}{2}$	93.9 "
1.00	96.6 "

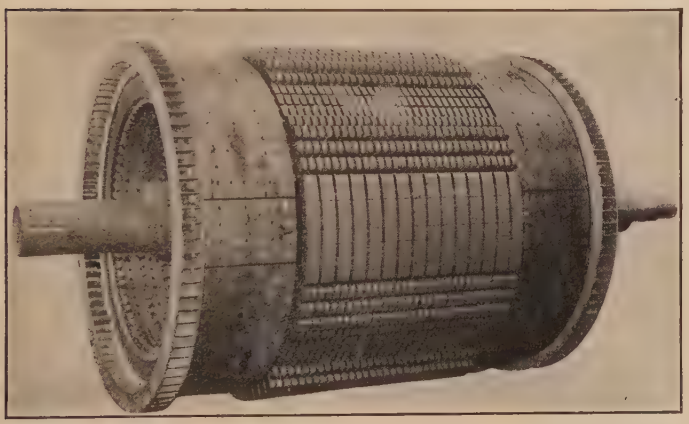


FIG. 61.—Rotor with Distributed Field Winding for a 4-pole, 15 000-kva., 25-cycle, 750 r.p.m., 3-phase Alternator, built by the General Electric Co. of America.

On pp. 94 to 96 the efficiencies of the 375-r.p.m. machine for this same rating were ascertained to be:

Load.	Efficiency.
$\frac{1}{4}$	88.2 per cent
$\frac{1}{2}$	93.1 "
1.00	95.6 "

For a 100-r.p.m., 2250-kw., 0.90-power-factor, 25-cycle design, the efficiencies would have been of the following order:

Load.	Efficiency.
$\frac{1}{4}$	88.0 per cent
$\frac{1}{2}$	92.6 "
1.00	94.6 "

The values may be brought together for comparison as follows:

EFFICIENCIES

Load.	100 r.p.m.	375 r.p.m.	3600 r.p.m.
$\frac{1}{4}$	88.0	88.2	88.5
$\frac{1}{2}$	92.6	93.1	93.9
1.00	94.6	95.6	96.6

While it is within the designer's power to modify the inherent tendencies corresponding to the rated speeds, nevertheless the values above set forth are representative. Thus while a full-load efficiency of 96.6 per cent is obtained for the 3600-r.p.m. design, the full-load efficiency of the 100-r.p.m. design is only 94.6 per cent. At quarter load the efficiency is practically as high for the 100-r.p.m. design as for the 3600-r.p.m. design. In judging of improvements in efficiency, it is the decrease in the percentage of losses which should be considered. Thus when the efficiency is increased from 94.6 per cent to 96.6 per cent, the losses are decreased from 5.4 per cent of the input to 3.4 per cent of the input. The decrease in the losses is thus:

$$\left(\frac{5.4-3.4}{5.4}\times 100=\right)37 \text{ per cent.}$$

In a polyphase generator driven by a steam turbine, it is now almost universal practice totally to enclose the generator, except so far as relates to the provision of suitable inlets and outlets for the circulating air, which is usually driven through the machine by means of fans located on the rotor. Consequently, in this type of machine, a step in the calculation relates to estimating the supply of air required suitably to limit the temperature

rise, and so to proportion the passages as to transmit the air in the quantities thus ascertained to be necessary.

In our 3600-r.p.m., 2250-kw. generator, the losses at full load amount to 80 kw. If the heat corresponding to this loss is to be carried away as fast as it is produced, then we must circulate sufficient air to abstract:

$$80 \text{ kw.-hr. per hr.}$$

A convenient starting point for our calculation is from the basis that:

$$1.16 \text{ w.-hr. raises 1 kg. of water } 1^\circ \text{ Cent.}$$

The specific heat of air is 0.24; that is to say, it requires only 0.24 times as much energy to raise 1 kg. of air by 1 degree Cent. as is required to raise 1 kg. of water by 1 degree Cent.

Consequently, to raise by 1 degree Cent., the temperature of 1 kg. of air, requires the absorption of:

$$1.16 \times 0.24 = 0.278 \text{ w.-hr.}$$

One kilogram of air at atmospheric pressure and at 30 degrees Cent. occupies a volume of 0.85 cu.m. Therefore, to raise 1 cu.m. of air by 1 degree Cent. requires:

$$\frac{0.278}{0.85} = 0.327 \text{ w.-hr.}$$

If, for the outgoing air, we assume a temperature 25 degrees above that of the ingoing air, then every cu.m. of air circulated through the machine will carry away:

$$0.327 \times 25 = 8.2 \text{ w.-hr.}$$

We must arrange for sufficient air to carry away:

$$80\,000 \text{ w.-hr. per hour.}$$

Consequently we must supply:

$$\frac{80\,000}{8.2} = 9800 \text{ cu. m. per hour;}$$

or:

$$\frac{9800}{60} = 163. \text{ cu. m. per minute.}$$

In dealing with the circulation of air it appears necessary to make the concession of employing other than metric units. We have:

$$1 \text{ cu.m.} = 35.4 \text{ cu.ft.}$$

Therefore in the case of our 2250-kw. generator, we must circulate:

$$163 \times 35.4 = 5800 \text{ cu.ft. per min.}$$

CHAPTER IV

THE DESIGN OF A POLYPHASE INDUCTION MOTOR WITH A SQUIRREL-CAGE ROTOR

The polyphase induction motor was brought to a commercial stage of development about twenty years ago. Many tens of thousands of such motors are now built every year. The design of polyphase induction motors has been the subject of many elaborate investigations and there has been placed at the disposal of engineers a large number of practical rules and data.

The design of such a motor may proceed from any one of many starting points and each designer has his preferred method. The author proposes to indicate the method which he has found to be the most useful for his purposes. It must not be inferred that any set of rules can be framed which will lead with certainty to the best design for any particular case. The most which can be expected is that the rules shall lead to a rough preliminary design which shall serve to fix ideas of the general orders of dimensions. Before he decides upon the final design, the enterprising designer will carry through a number of alternative calculations in which he will deviate in various directions from the original design. A consideration of the several alternative results at which he will thus arrive, will gradually lead him to the most suitable design for the case which he has in hand.

The method of design will be expounded in the course of working through an illustrative example.

ILLUSTRATIVE EXAMPLE

Let it be required that a three-phase squirrel-cage induction motor be designed. The normal rating is to be 200 hp. and the motor is to be operated from a 1000-volt, 25-cycle circuit. It is desired that its speed shall be in the close neighborhood of 500 r.p.m.

Determination of the Number of Poles. Denoting the speed in revolutions per minute by R, then the speed in revolutions per second is equal to $\frac{R}{60}$. If we denote the number of poles by P, and the periodicity in cycles per second by $\sim$, then we have:

$$\sim = \frac{R}{60} \times \frac{P}{2}.$$

In our case we have:

$$\sim = 25 \qquad R = 500.$$

Therefore:

$$P = \frac{2 \times 60 \times 25}{500} = 6.$$

If we design the motor with 6 poles, the "synchronous" speed will be 500 r.p.m. At no-load, the motor runs at practically its "synchronous" speed; that is to say, its "slip" is practically zero. The term "slip" is employed to denote the amount by which the actual speed of the motor is less than the "synchronous" speed. An appropriate value for the slip of our motor at its rated load, is some 2 per cent, or even less. Taking it for the moment as 2 per cent, we find that, at rated load, the speed will be $500 - 0.02 \times 500 = 490$ r.p.m.

The variations in the speed between no load and full load are so slight that at many steps in the calculations the speed may be taken at the approximate value of 500 r.p.m., thus avoiding superfluous refinements which would merely complicate the calculations and serve no useful purpose.

Rated Output Expressed in Watts. Since one horse-power is equal to 746 watts, the rated output of our 200-h.p. motor may also be expressed as $200 \times 746 = 149\,200$ watts.

Determination of τ, the Polar Pitch. The distance (in cm.) measured at the inner circumference of the stator, from the center of one pole to the center of the next adjacent pole, is termed the "polar pitch" and is denoted by the letter τ. Rough pre-

liminary values for τ are given in Table 11 for designs for 25 and 50 cycles for a wide range of outputs and speeds. These values have been found by experience to be appropriate.

TABLE 11.—PRELIMINARY VALUES FOR τ (THE POLAR PITCH) FOR USE IN DESIGNING THREE-PHASE INDUCTION MOTORS

Rated Output.		25 Cycles.					
H.P.	Watts.	$P=4$. $R=750$.	$P=6$. $R=500$.	$P=8$. $R=375$.	$P=10$. $R=300$.	$P=12$. $R=250$.	$P=14$. $R=218$.
5	3720	17.0	14.0	12.0			
10	7460	19.0	16.0	14.0	13.0		
20	14900	22.0	19.0	17.0	15.5	14.0	
40	29900	26.0	22.0	20.0	18.0	17.0	16.0
60	44700	28.0	24.0	22.0	20.0	19.0	18.0
80	59700	30.0	26.0	24.0	21.5	20.5	19.0
100	74600	31.5	27.0	24.5	23.0	21.5	20.0
150	112000	35.0	30.0	27.0	25.0	23.5	22.0
200	149000	38.0	32.5	29.5	27.5	26.0	24.0
300	224000	43.0	37.0	33.0	30.5	29.0	27.0
400	299000	47.0	40.0	36.0	33.0	31.0	29.0
500	373000	50.0	42.0	37.5	34.5	32.5	30.0

Rated Output		50 Cycles.									
H.P.	Watts	$P=4$ $R=1500$	$P=6$ $R=1000$	$P=8$ $R=750$	$P=10$ $R=600$	$P=12$ $R=500$	$P=14$ $R=429$	$P=16$ $R=375$	$P=20$ $R=300$	$P=24$ $R=250$	$P=28$ $R=214$
5	3720	16.0	13.0	11.0							
10	7460	17.0	14.0	12.0	11.0						
20	14900	19.0	16.0	14.0	13.0	12.0	11.5				
40	29900	22.0	19.0	17.0	15.5	14.5	13.5	13.0			
60	44700	24.0	20.5	18.5	17.0	16.0	15.0	14.5	13.5		
80	59700	25.5	22.0	19.5	18.0	17.0	16.0	15.5	14.5	13.5	
100	74600	27.0	23.0	20.5	19.0	18.0	17.0	16.0	15.0	14.0	13.5
150	112000	29.5	25.0	22.5	21.0	20.0	18.5	17.5	16.5	15.5	15.0
200	149000	31.5	27.0	24.5	23.0	21.5	20.5	19.5	18.0	17.0	16.9
300	224000	35.0	30.0	27.0	25.0	24.0	22.5	21.5	20.0	18.5	17.5
400	299000	38.0	32.5	29.5	27.0	25.5	24.0	23.0	21.5	20.0	19.0
500	373000	40.0	34.5	31.5	29.0	27.0	25.5	24.0	22.5	21.5	20.5

For our 200-h.p., 6-pole, 25-cycle design, we find from Table 11, the value:

$$\tau = 32.5 \text{ cm.}$$

THE OUTPUT COEFFICIENT

The next step relates to the determination of a suitable value for ξ, the "Output Coefficient," which is defined by the following formula:

$$\xi = \frac{W}{D^2 \times \lambda g \times R},$$

in which

W = Rated output in watts, (which is equal to 746 times the rated output in h.p.),
D = Diameter at air-gap, in decimeters, i.e., the internal diameter of the stator,
λg = Gross core length, in decimeters,
R = rated speed, in revolutions per minute.

As in the earlier chapters of this treatise, the symbols D and λg will sometimes be employed for denoting respectively the air-gap diameter and the gross core length, expressed, as above, in decimeters, but more usually they will denote these quantities as expressed in centimeters. The student can soon accustom himself to distinguishing, from the magnitudes of these quantities, whether decimeters or centimeters are intended, and thus will not experience any difficulty of consequence, in this double use of the same symbols.

Values of ξ suitable for preliminary assumptions are given in Table 12. In this table, ξ is given as a function of P and τ.

For our design we have:

$$P = 6, \quad \tau = 32.5 \text{ cm}.$$

The corresponding value of ξ in Table 12 is about 2.0. But let us for our design be satisfied with a less exacting value and take:

$$\xi = 1.80.$$

Therefore:

$$1.80 = \frac{149\,200}{D^2 \times \lambda g \times 500};$$

$$D^2 \lambda g = \frac{149\,200}{1.80 \times 500} = 166.$$

TABLE 12.—PRELIMINARY VALUES FOR ξ (THE OUTPUT COEFFICIENT) FOR USE IN DESIGNING THREE-PHASE SQUIRREL-CAGE INDUCTION MOTORS.

(The figures at the heads of the vertical columns give the numbers of poles.)

τ, the polar pitch (in cm.).	4.	6.	8.	10.	12.	14.	16	18.	20.	22.	24.
11		0.80	1.07	1.12	1.15	1.14	1.12	1.11	1.10	1.08	1.05
12		0.95	1.14	1.19	1.21	1.20	1.20	1.18	1.16	1.14	1.12
14		1.13	1.28	1.30	1.31	1.30	1.28	1.26	1.26	1.25	1.25
16	0.85	1.28	1.40	1.41	1.41	1.41	1.40	1.40	1.40	1.40	1.40
18	1.11	1.40	1.50	1.50	1.51	1.51	1.51	1.51	1.51	1.51	1.51
20	1.27	1.52	1.59	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60
22	1.42	1.63	1.69	1.70	1.70	1.70	1.70	1.70	1.70	1.69	1.69
24	1.53	1.72	1.78	1.79	1.80	1.80	1.80	1.80	1.80	1.80	1.80
26	1.63	1.82	1.86	1.87	1.87	1.87	1.88	1.88	1.88	1.88	1.88
28	1.71	1.90	1.93	1.93	1.94	1.94	1.95	1.95	1.95	1.95	1.95
30	1.79	1.97	1.99	2.00	2.00	2.01	2.01	2.01	2.02	2.02	2.02
35	1.90	2.08	2.10	2.11	2.11	2.11	2.12	2.12	2.12	2.12	2.12
40	1.98	2.17	2.20	2.20	2.21	2.21	2.22				
45	2.04	2.21	2.26	2.28							
50	2.07	2.23	2.30								

But D (in cm.), is equal to:

$$\frac{P \times \tau}{\pi}$$

Therefore:

$$D = \frac{6 \times 32.5}{\pi} = 62.0 \text{ cm.} = 6.20 \text{ dm.}$$

Therefore:

$$\lambda g \text{ (in dm.)} = \frac{166}{6.20^2} = 4.32 \text{ dm. or } 43.0 \text{ cm.}$$

D and λg and τ are the three characteristic dimensions of the design with which we are dealing.

$D^2\lambda g$ (with D and λg expressed in decimeters), is also, in itself, a useful value to obtain at an early stage of the calculation of a design. We have, for our motor;

$$D^2\lambda g = 166.$$

PRELIMINARY ESTIMATE OF THE TOTAL NET WEIGHT

A rough preliminary idea of the total net weight of an induction motor may be obtained from a knowledge of its $D^2\lambda g$. The "Total Net Weight" may be taken as the weight exclusive of slide rails and pulley. In Table 13, are given rough representative values for the Total Net Weights of induction motors with various values of $D^2\lambda g$.

TABLE 13.—VALUES OF THE TOTAL NET WEIGHT OF INDUCTION MOTORS.

$D^2\lambda g$.	Total Net Weight in Metric Tons, (i.e., in Tons of 2204 Lbs.).
10	0.27
20	0.40
40	0.73
60	0.98
80	1.20
100	1.40
150	1.90
200	2.30
250	2.70
300	3.00
350	3.25
400	3.45

PRELIMINARY ESTIMATE OF THE TOTAL WORKS COST

The Total Works Cost will necessarily also be a very indefinite quantity, susceptible to large variations with variations in the proportions and arrangement of a design. It is even more greatly dependent upon the equipment and management of the Works at which the motor is manufactured. Neverthless, some rough indication appropriate for squirrel-cage induction motors is afforded by the data in Table 14.

TABLE 14.—TOTAL WORKS COST OF SQUIRREL-CAGE INDUCTION MOTORS.

Total Net weight of Motor in Metric Tons.	Total Works Cost per Ton, in Dollars.
0.20	310
0.40	300
0.60	290
0.80	285
1.00	280
1.50	270
2.00	260
2.50	250
3.00	240
3.50	230
4.00	225

For our 200-h.p. motor, we have $D^2\lambda g = 166$.

From Table 13, we ascertain that the Total Net Weight is some 2.00 tons. From Table 14, it is found that the Total Works Cost is of the order of \$260. per ton. Consequently we have Total Works Cost $= 2.00 \times 260 = \$520$.

ALTERNATIVE METHOD OF ESTIMATING THE TOTAL WORKS COST

An alternative method of estimating the T.W.C. of an induction motor is based on the following formula:

$$TWC \text{ (in dollars)} = K \times D \times (\lambda g + 0.7\tau),$$

where D, λg, and τ are given in centimeters. K is obtained from Table 15.

TABLE 15—VALUES OF K IN FORMULA FOR T. W. C.

Air-gap Diameter, D. in Centimeters.	Values of K.
10	0.098
20	0.105
40	0.112
60	0.120
80	0.128
100	0.133
150	0.140
200	0.148

In the 200-h.p. motor which is serving us as an example, we have:

$$\tau = 32.5, \qquad 0.7\tau = 22.7,$$

$$\lambda g = 43.0, \qquad \lambda g + 0.7\tau = 65.7,$$

$$D = 62.0, \qquad K \text{ (from Table 15)} = 0.122,$$

$$TWC = 0.122 \times 62 \times 65.7 = \$508.$$

Thus the results by the two methods are \$520 and \$508 which are in good agreement with each other. Usually the agreement will be far less close and an average of the two values is preferable as a guide. It is interesting to note that the TWC per rated horse-power is, in the case of this motor, some:

$$\frac{514}{200} = \$2.57.$$

The motor could not, of course, be **bought** at any such price, for the TWC merely covers all the costs incurred up to the delivery of the completed machine to the shipping department, to be packed. Selling and shipping expenses, as also profits, must be added to the TWC to arrive at the price at which the

machine should be sold and it is not unusual to find the selling price over double the Total Works Cost. This is one of the penalties of the competitive system of supplying the needs of mankind.

The Peripheral Speed. It is well, before proceeding further with the design, to calculate the peripheral speed. Let us denote by S, the peripheral speed, expressed in meters per second.

$$S = \frac{\pi \times D}{100} \times \frac{R}{60}.$$

For our design:

$$S = \frac{\pi \times 62 \times 500}{100 \times 60} = 16.2 \text{ meters per second.}$$

In this instance the peripheral speed is very low, and does not constitute a limiting consideration from the point of view of mechanical strength. For other speeds, ratings, and periodicities, the preliminary data as derived from the rules which have been set forth, might lead to an undesirably-high peripheral speed. Consequently it is well to ascertain the peripheral speed at an early stage of the calculations and arrange to reduce D and τ in cases where the electrical design ought to be sacrificed in some measure in the interests of improving the mechanical design.

PERIPHERAL LOADING

We shall next deal with the determination of the number of conductors to be employed. The product of the number of conductors and the current per conductor, (i.e., the ampere-conductors), constitutes a quantity to which the term " peripheral loading " may be applied. Designers find from experience that it is desirable to employ certain definite ranges of values for the peripheral loading per centimeter of periphery, measured at the air-gap. In Table 16, are given values which will serve as preliminary assumptions for a trial design.

TABLE 16.—PRELIMINARY ASSUMPTIONS FOR THE PERIPHERAL LOADING OF AN INDUCTION MOTOR.

D (in Centimeters).	Stator Ampere Conductors per Centimeter of Gap Periphery.	
	25 Cycles.	50 Cycles.
15	140	180
20	180	220
25	220	270
30	270	310
40	320	350
60	370	400
80	380	420
100	390	430
120	400	440

For motors wound for *very low* pressures, (say 250 volts), somewhat higher values may be employed, especially in the larger sizes. On the contrary, for motors wound for *high* pressures—say 2500 volts or more—it is necessary to employ for the ampere conductors per centimeter of periphery, lower values than those indicated in the table, especially in motors of very small diameter. Indeed the loss of space in providing for slot insulation renders it very undesirable to wind small motors for very high pressures. It is better, in such cases, to interpose step-down transformers between the supply system and the motor or motors.

Experienced designers will find occasions where the periphery may, and **should,** be loaded with ampere conductors much more highly than corresponds with the data in Table 16. On the other hand, there are often difficult ratings, (as regards overload capacity and other features which will later come in for consideration), where the peripheral loading should (and **must**) be much lower than the values indicated in Table 16. As a matter of fact, the rated speed and output, the periodicity and pressure, and also the stipulated instantaneous overload capacity, all require to be taken into consideration. But at this early stage in the design, it is useful to take a value from Table 16. For our case, D is equal to 62, and the corresponding value for the peripheral loading is seen to be 372 ampere conductors per centimeter.

Consequently the total peripheral loading is:

$$62 \times \pi \times 372 = 72\,500 \text{ ampere-conductors;}$$

or

$$\frac{72\,500}{3} = 24\,200 \text{ ampere-conductors per phase.}$$

From this product we wish next to segregate the ampeers and the conductors. We may do this by dividing the ampere-conductors by the full-load current per phase. We cannot estimate the precise value of the full-load current per phase until the design has been completed, as its precise value depends upon the efficiency and power-factor at full load. But we cannot complete the design without determining upon the suitable number of conductors to employ. Hence it becomes necessary to have recourse to tables of rough approximate values for the full-load efficiency and the power-factor of designs of various ratings. Such values are given in Tables 17 and 18.

EFFICIENCY AND POWER-FACTOR

For the case of squirrel-cage induction motors for moderate pressures, and in the absence of any specially exacting requirements as regards capacity for carrying large instantaneous overloads, we may proceed from the basis of the rough indications in Tables 17 and 18. From these tables we obtain:

Full-load efficiency = 91 per cent.
Full-load power-factor = 0.91.

The required estimation of the full-load current may be carried out as follows:

Horse-power output at rated load	$= 200$
Watts output at rated load	$= 200 \times 746 = 149\,200$
Efficiency at rated load	$= 0.91$
Watts input at rated load	$= \frac{149\,200}{0.91} = 164\,000$

Watts input *per phase* at rated load $= \frac{164\,000}{3} = 54\,700$

Power-factor at rated load $= 0.91$

Volt-amp. input per phase at rated load $= \frac{54\,700}{0.91} = 60\,200$

Pressure between terminals (in volts) $= 1000$

Phase pressure $= \frac{1000}{\sqrt{3}} = 577$

Current per phase at rated load $= \frac{60\,200}{577} = 104$

TABLE 17.—PRELIMINARY VALUES OF FULL LOAD EFFICIENCY, IN PER CENT, FOR POLYPHASE SQUIRREL-CAGE INDUCTION MOTORS. THE VALUES GIVEN CORRESPOND TO THOSE OF NORMAL MOTORS.

Rated Output in Horse-power.	Efficiencies for the Following Periodicities and Synchronous Speeds.								
	Periodicity = 12.5 Cycles.			Periodicity = 25 Cycles.			Periodicity = 50 Cycles.		
	375 r.p.m.	188 r.p.m.	94 r.p.m.	750 r.p.m.	375 r.p.m.	188 r.p.m.	1500 r.p.m.	750 r.p.m.	375 r.p.m.
10	76.7	72.8	65.0	82.5	79.1	74.6	84.1	80.6	75.7
20	79.0	74.7	69.4	84.0	80.6	76.6	85.5	82.6	78.1
30	80.7	77.3	72.3	85.0	82.0	78.1	87.3	84.1	80.4
40	81.4	78.0	73.5	86.5	83.3	79.8	88.8	86.0	82.1
50	83.0	80.0	75.6	87.5	84.5	81.1	90.0	87.3	84.0
60	84.0	81.1	77.1	88.0	85.5	82.5	90.8	88.3	85.3
80	86.0	83.3	80.1	89.8	87.3	84.8	92.5	90.3	87.8
100	87.0	85.0	82.1	91.3	88.8	86.5	93.8	91.8	89.3
300	89.0	87.0	84.8	92.5	90.8	88.8	95.0	93.5	90.8
500	89.8	88.0	86.0	93.2	91.5	89.8	95.5	94.0	91.8
700	90.5	88.8	87.0	93.8	92.3	90.5	95.7	94.3	92.3
1000	91.0	89.8	88.5	94.0	92.8	91.3	95.8	95.5	92.8

Conductors per phase. For a preliminary estimate we may proceed as follows:

Conductors per phase $= \frac{24\,200}{104} = 232.$

Conductors per pole per phase $= \frac{232}{6} = 38.7.$

We must round off this value to some suitable whole number, say 39 or 40, taking whichever leads to the best arrangement of the winding. An inspection indicates that we should consider the following alternatives:

40 conductors arranged 8 per slot in $\left(\frac{40}{8}=\right)$ 5 slots.

40 " " 10 " " $\left(\frac{40}{10}=\right)$ 4 "

39 " " 13 " " $\left(\frac{39}{13}=\right)$ 3 "

40 " " 20 " " $\left(\frac{40}{20}=\right)$ 2 "

TABLE 18.—PRELIMINARY VALUES FOR FULL-LOAD POWER FACTOR OF POLYPHASE SQUIRREL-CAGE INDUCTION MOTORS OF NORMAL DESIGN.

Rated Output in Horse-power.	Power-factor for the Following Periodicities and Synchronous Speeds.								
	Periodicity = 12.5 Cycles.			Periodicity = 25 Cycles.			Periodicity = 50 Cycles.		
	375 r.p.m.	188 r.p.m.	94 r.p.m.	750 r.p.m.	375 r.p.m.	188 r.p.m.	1500 r.p.m.	750 r.p.m.	375 r.p.m
10	0.896	0.820	0.770	0.882	0.820	0.760	0.898	0.830	0.750
20	0.906	0.829	0.780	0.892	0.830	0.770	0.904	0.840	0.760
30	0.916	0.838	0.790	0.900	0.837	0.780	0.908	0.847	0.770
40	0.921	0.847	0.800	0.906	0.844	0.790	0.914	0.852	0.780
50	0.928	0.855	0.810	0.911	0.850	0.800	0.917	0.857	0.790
60	0.931	0.864	0.820	0.916	0.856	0.810	0.921	0.862	0.800
80	0.941	0.872	0.830	0.924	0.862	0.820	0.926	0.867	0.810
100	0.947	0.880	0.840	0.930	0.871	0.830	0.928	0.872	0.820
300	0.955	0.887	0.850	0.938	0.880	0.840	0.931	0.876	0.830
500	0.960	0.896	0.860	0.942	0.886	0.850	0.932	0.880	0.840
700	0.962	0.904	0.870	0.945	0.892	0.860	0.932	0.884	0.850
1000	0.963	0.912	0.880	0.946	0.900	0.870	0.932	0.888	0.860

Number of Slots per Pole per Phase. There cannot be given any absolute rule as regards the number of slots per pole per phase which should be employed. In a general way it may be

stated that the quality of the performance of the motor is higher, the greater the number of slots per pole per phase. But the overall dimensions and the weight and the Total Works Cost increase with increasing subdivision of the winding amongst many slots, and consequently the designer should endeavor to arrive at a reasonable compromise between quality and cost.

The Slot Pitch. We may designate as the slot pitch the distance (measured at the air-gap) from the center line of one slot to the center line of the next adjacent slot. Since this quantity is usually small, it is generally convenient to express it in mm. Good representative values for the stator slot pitch are given in Table 19. The values in the table may be taken as applying to designs for moderate pressures. The higher the pressure, the more must one depart from the tabulated values in the direction of employing fewer slots.

TABLE 19.—VALUES OF STATOR SLOT PITCH FOR INDUCTION MOTORS.

τ, the Polar Pitch (in cm.).	Stator Slot Pitch, (in mm.)
12	15.0
14	16.4
16	17.7
18	18.9
20	20.0
25	22.3
30	24.4
35	26.0
40	27.0

From Table 19 we select as appropriate for our motor, a trial slot pitch of about 25 mm. Since τ is equal to 325 mm., we should have:

$$\frac{325}{25} = 13 \text{ slots per pole.}$$

But the number finally chosen should be a multiple of 3, the number of phases. Consequently let us employ 12 slots per pole,

or $\left(\frac{12}{3}=\right)$ 4 slots per pole per phase (*pppp*). Thus we must revise the stator slot pitch to:

$$\frac{325}{12}=27.1 \text{ mm.}$$

The Total Number of Stator Slots. Since our machine has 6 poles, we shall have:

$$12\times 6=72 \text{ stator slots.}$$

The appropriate slot layout will be based on 10 conductors per slot and:

$$4\times 10=40 \text{ conductors pppp.}$$

and

$$\frac{6\times 40}{2}=120 \text{ turns in series per phase.}$$

Let us denote the turns in series per phase by T. Then:

$$T=120.$$

THE PRESSURE FORMULA

In our discussion of the design of generators of alternating electricity we have become acquainted with the pressure formula:

$$V=K\times T\times \sim \times M.$$

In this formula we have:

V = the phase pressure in volts;
K = a coefficient;
T = turns in series per phase;
$\sim$ = periodicity in cycles per second;
M = flux per pole in megalines.

For a *motor*, the phase pressure in the above formula must, for full load, be taken smaller than the terminal pressure, to the extent of the IR drop in the stator windings. But at no load

the phase pressure is equal to the terminal pressure divided by $\sqrt{3}$.

Therefore,

$$\text{Phase pressure} = \frac{1000}{1.732} = 577 \text{ volts.}$$

The coefficient K depends upon the spread of the winding and the manner of distribution of the flux. For the conditions pertaining to a three-phase induction motor with a full-pitch winding we have:

$$K = 0.042.$$

For other winding pitches, the appropriate value of K may be derived by following the rule previously set forth on pp. 16 to 18 of Chapter II, where the voltage formula for generators of alternating electricity is discussed.

Since our motor is for operation on a 25-cycle circuit, we have:

$$\sim = 25.$$

Thus at no load we have:

$$577 = 0.042 \times 120 \times 25 \times M;$$

$$M = 4.57 \text{ megalines.}$$

THE MAGNETIC CIRCUIT OF THE INDUCTION MOTOR

In Fig. 62 are indicated the paths followed by the magnetic lines in induction motors with 2, 4, and 8 poles. One object of the three diagrams has been to draw attention to the dependence of the length of the iron part of the path, on the number of poles. Thus while in the 2-pole machine, some of the lines extend over nearly a semi-circumference, in the stator core and in the rotor core; their extent is very small in the 8-pole design. As a consequence, the sum of the magnetic reluctances of the air-gap and teeth constitutes a greater percentage of the total magnetic reluctance, the greater the number of poles.

Fig. 63 has been drawn to distinguish that portion of the total flux which corresponds to one pole. It is drawn to correspond to a 6-pole machine.

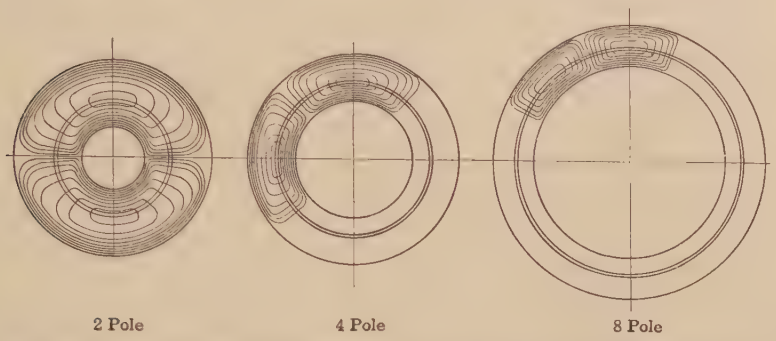


Fig. 62.—Diagrammatic Sketches of 2-, 4-, and 8-pole Induction-motor Cores, showing the Difference in the Mean Length of the Magnetic Path.

At this stage of our calculations, we wish to ascertain the cross-section which must be allowed for the stator teeth. A crest density of 15 500 lines per sq. cm. is appropriate for the stator

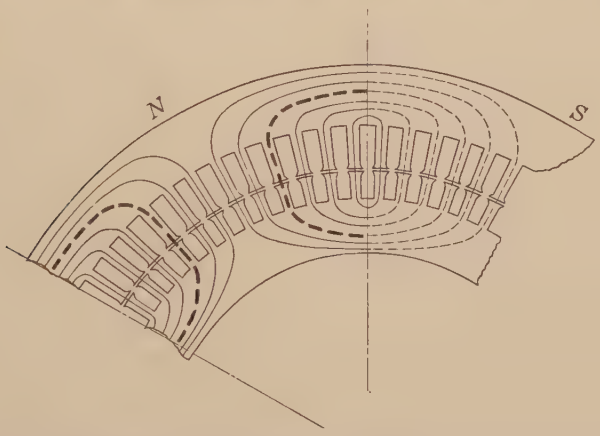
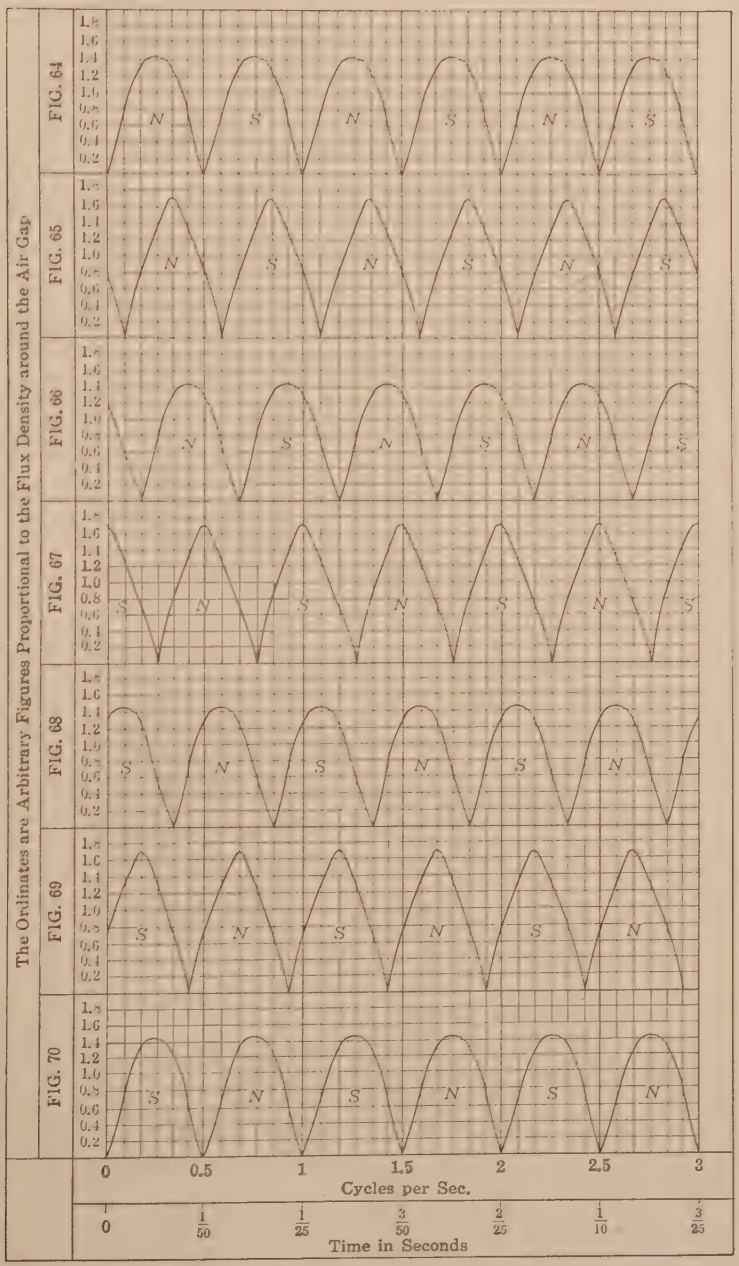


Fig. 63.—Diagrammatic Representation of that Portion of the Total Path which Corresponds to One Pole of an Induction Motor. [The heavy dotted lines indicate the mean length of the magnetic path for one pole.]

teeth in such a design as that which we are considering. A skilled designer will, on occasions, resort to tooth densities as high as 19 000, but it requires experience to distinguish appropriate cases

for such high densities and the student will be well advised to employ lower densities until by dint of practice in designing, he is competent to exercise judgment in the matter. In general the designer will employ a lower tooth density the greater the number of poles. This is for two reasons, firstly, as already mentioned in connection with Fig. 62, the magnetic reluctance of the air-gap and teeth constitutes a greater percentage of the total reluctance the greater the number of poles; and secondly, (for reasons which will be better understood at a later stage), a high tooth density acts to impair the power-factor of a machine with **many** poles, to a greater extent than in the case of a machine with **few** poles.

In Figs. 64 and 65 are shown two diagrams. These represent the distribution of the flux around the periphery of our 6-pole motor at two instants one-twelfth of a cycle apart. Since the periodicity is 25 cycles per second, one-twelfth of a cycle occupies $\left(\frac{1}{25 \times 12} = \right)$ $\frac{1}{300}$th of a second. After another $\frac{1}{300}$th of a second the flux again assumes the shape indicated in Fig. 64, but displaced further along the circumference, as indicated in Fig. 66. In other words, as the flux travels around the stator core, its distribution is continually altering in shape from the typical form shown in Fig. 64, to that shown in Fig. 65, and back to that shown in Fig. 66 (which is identical with Fig. 64, except that it has advanced further in its travel around the stator). Successive positions of the flux, each $\frac{1}{300}$th second later than its predecessor, are drawn in Figs. 67 to 70. Comparing Fig. 70 with Fig. 64, we see that they are identical except that in Fig. 64, a south flux occupies those portions of the stator, which, in Fig. 64, were occupied by a north flux. In other words, a half cycle has occurred in the course of the $\frac{1}{300} \times \frac{1}{2} = \frac{1}{600}$th of a second which has elapsed while the flux has traveled from the position shown in Fig. 64, to that shown in Fig. 70. A **whole** cycle will have occurred in $\frac{1}{25}$th of a second (the periodicity is 25 cycles per second); and the flux will then have been displaced to the extent of the space occupied by one *pair* of poles. At the end of the time occupied by 3 cycles ($\frac{3}{25}$ths of a second) the flux will have completed one revolution around the stator core, since the machine has $\left(\frac{6}{2} = \right)$ 3 *pairs* of poles.



FIGS. 64 to 70.—Diagrams Indicating the Variations in the Shape of the Flux as it Progresses Around the Stator Core.

In estimating the magnetomotive force (mmf.) which must be provided for overcoming the reluctance of the magnetic circuit, we must base our calculations on the **crest** flux density. This corresponds to the flux distributions represented in Figs. 65, 67 and 69. It can be shown * that the crest density indicated in these figures is 1.7 times the average density. In other

FIG. 71.—Diagram Illustrating that the *Crest* Density in the Air-gap and Teeth of an Induction Motor is 1.7 Times the *Average* Density.

words, the crest density with the flux distribution corresponding to the peaked curve in Fig. 71, is 1.7 times the average

* This is demonstrated, step by step, on pp. 380 to 390 of the 2d edition of the author's "Electric Motors" (Whittaker & Co., London and New York, 1910).

density indicated by the rectangle in the same figure. Each tooth is, in turn, located at the center of the rotating flux and is in turn subjected to this crest density.

Thus for our assumption of a crest density of 15 500 lines per square centimeter we shall have:

$$\text{Average density in stator teeth} = \frac{15\,500}{1.7} = 9100 \text{ lines per sq.cm.}$$

Since we have a flux of 4.57 megalines, we must provide, per pole, a tooth cross-section of:

$$\frac{4\,570\,000}{9100} = 500 \text{ sq.cm.}$$

There are 12 stator teeth per pole. The cross-section of each tooth must thus be:

$$\frac{500}{12} = 41.6 \text{ sq.cm.}$$

Before we can attain our present object of determining upon the width of the tooth, we shall have to digress and take up the matter of the proportioning of the ventilating ducts.

Ventilating Ducts

The employment of a large number of ventilating ducts in the cores of induction motors, renders permissible, from the temperature standpoint, the adoption of much higher flux densities and current densities than could otherwise be employed, and thus leads to a light and economical design.

In Table 20 are given rough values for the number of ducts, each 15 mm. wide, which may be taken as suitable, under various circumstances of peripheral speed and values of λg.

In our case, where the peripheral speed is 16.2 meters per second and λg is 43, the table indicates 1.8 ducts per dm., or a total of

$1.8 \times 4.3 = 7.7$ ducts, to be a suitable value. Eight ducts will be employed and they will require $8 \times 1.5 = 12.0$ cm.

TABLE 20.—VENTILATING DUCTS FOR INDUCTION MOTORS.

Peripheral Speed in Meters per Second.	Number of Ventilating Ducts (Each 15 mm. Wide), which Can Appropriately be Used, per Decimeter of λg.		
	$\lambda g = 10$.	$\lambda g = 30$.	$\lambda g = 50$.
10	2.2	2.3	2.4
15	1.7	1.9	2.1
20	1.5	1.7	1.9
25	1.3	1.5	1.7
30	1.1	1.3	1.5
35	1.0	1.2	1.3
40	0.9	1.1	1.2

The varnish by means of which adjacent core plates are insulated from one another, will occupy some 10 per cent of the total depth occupied by the insulated core plates.

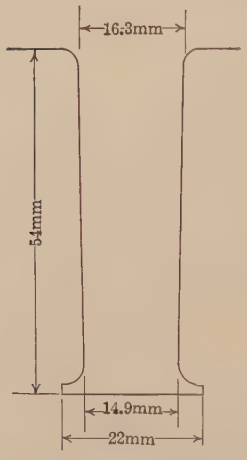


FIG. 72.—Stator Tooth of 200 H.P. Induction Motor.

Thus for λn, the net core length, we arrive at the value:

$$\lambda n = (43 - 12) \times 0.9 = 31 \times 0.9 = 27.9 \text{ cm.}$$

WIDTH OF STATOR TOOTH

We may now complete the calculation of the width of the stator tooth at its narrowest part. The tooth will be of the form indicated in Fig. 72, and the narrowest part (or neck) will be at a diameter not appreciably greater than the air-gap diameter. It is not important to be specially exact in the matter, so let us make the rough preliminary assumption that the diameter at this narrowest part is 1 cm. greater than D, the air-gap diameter. We have:

$$D = 62.0 \text{ cm.}$$

Diameter to neck of tooth $= 62.0 + 1.0 = 63.0$ cm.

Tooth pitch at neck $= \frac{630\pi}{72} = 27.5$ mm.

Required cross-section of tooth at neck $= 41.6$ sq.cm.

Width of tooth at neck $= \frac{4160}{\lambda n} = \frac{4160}{279} = 14.9$ mm.

Width of Slot. The slot will have parallel sides, and its width will be:

$$27.5 - 14.9 = 12.6 \text{ mm.}$$

This will be its width **when punched.** Owing to inevitable slight inaccuracies in building up the stator core from the individual punchings, the **assembled** width of the slot will be some 0.3 mm. less, or

$$12.6 - 0.3 = 12.3 \text{ mm.}$$

This allowance of 0.3 mm. is termed the "**slot tolerance.**"

Having now determined the width of the stator slot, it would appear in order to proceed at once to determine its depth. But this depends upon the copper contents for which space must be provided. Consequently we must now turn our attention to the determination of:

The Dimensions of the Stator Conductor. The current density in the stator conductor is determined upon as a compromise amongst a number of considerations, one of the chief of which is the permissible value of the watts per square decimeter (sq.dm.) of peripheral radiating surface at the air-gap. This value is itself influenced by such factors as the peripheral speed and the ventilating facilities provided, hence the current density will also be influenced by these considerations.

The value of the watts per square decimeter of peripheral radiating surface at the air-gap, cannot, unfortunately, be ascertained until a later stage when we shall have determined not only the copper losses, but also the core loss. If, at that later stage, the value obtained for the watts per square decimeter of peripheral radiating surface at the air-gap, shall be found to be unsuitable, it will be necessary to readjust the design.

Table 21 has been compiled to give preliminary representative values for the stator current density for various outputs and peripheral speeds, for designs of normal proportions.

TABLE 21.—PRELIMINARY VALUES FOR THE STATOR CURRENT DENSITY.

Rated Output in h.p.	Current Density for Various Peripheral Speeds in Meters per Second (mps).			
	10 mps.	20 mps.	30 mps.	40 mps.
5	400			
10	380	400		
50	350	370	390	
100	320	340	360	380
500	290	320	340	350
1000		280	300	310

The Slot Insulation. The fact that the slot has an assembled width of 12.3 mm. is not to be taken as indicating that this width is available for the conductors. The iron core must be separated from the conductors by a considerable thickness of insulation. This insulation consists preferably in specially-manufactured tubes of high-grade insulating material. Suitable thicknesses are indicated in Table 22.

TABLE 22.—VALUES FOR THE THICKNESS OF THE STATOR SLOT LINING.

Normal Pressure (in Volts) for Which the Induction Motor is Wound.	Thickness of Slot Lining in mm.
500	0.9
1 000	1.4
2 000	2.3
3 000	2.9
4 000	3.3
6 000	4.0
8 000	4.7
10 000	5.2
12 000	5.6

In Fig. 73 a sketch is given of the slot with the insulating tube in place, and before winding. The precise depth has not

yet been determined, but the width of the available space is seen to be $12.3-2\times1.4=9.5$ mm. Thus we see that a space 9.5 mm. wide is available for the insulated conductors.

From Table 21 we ascertain that the stator conductor should be proportioned for about 300 amperes per square centimeter. Since we have, for the full-load current, $I=104$ amperes, the cross-section should be:

$$\frac{104}{300}=0.347 \text{ sq.cm. or } 34.7 \text{ sq.mm.}$$

In the interests of securing a suitable amount of flexibility in the process of winding, let us divide this aggregate cross-section into two conductors which shall be in parallel and each of which shall have a cross-section of some:

$$\frac{34.7}{2}=17.4 \text{ sq.mm.}$$

The diameter of a wire with a cross-section of 17.4 sq.mm. is:

$$D=\sqrt{\frac{4\times17.4}{\pi}}=4.70 \text{ mm.}$$

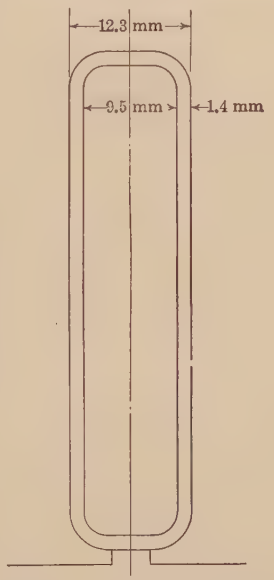


FIG. 73.—Stator Slot of 200 H.P. Induction Motor, showing Insulating Tube in Place.

The bare diameter of each wire would, on this basis, be 4.70 mm.

In Table 23 are given the thicknesses of insulation on suitable grades of cotton-covered wires employed in work of this nature.

We see that our wire of 4.70 mm. diameter would, if double cotton covered, have a thickness of insulation of about 0.18 mm. Consequently its insulated diameter would be:

$$4.70+2\times0.18=4.70+0.36=5.06 \text{ mm.}$$

But the width of the winding space is seen from Fig. 73 to be only 9.5 mm. The natural arrangement in this case, would be

to place the two components side by side. Consequently the insulated diameter must not exceed:

$$\frac{9.5}{2} = 4.75 \text{ mm.}$$

TABLE 23.—VALUES OF THE THICKNESS OF COTTON COVERING.

Diameter of Bare Conductor (in mm.).	Thickness of Insulation (in mm.).		
	Single Cotton Covered.	Double Cotton Covered.	Triple Cotton Covered.
1	0.060	0.100	
2	0.080	0.127	0.180
3	0.098	0.150	0.207
4	0.112	0.167	0.227
5	0.123	0.183	0.244
6	0.133	0.196	0.258
8	0.147	0.214	0.279
10		0.220	0.288
12			0.290

So let us reduce the *bare* diameter to:

$$4.75 - 0.36 = 4.39 \text{ mm.}$$

The readjusted conductor has a cross-section of only:

$$\frac{\pi}{4} \times 4.39^2 = 15.1 \text{ sq.mm.}$$

The two component conductors make up a cross-section of:

$$2 \times 0.151 = 0.302 \text{ sq.cm.}$$

The revised current density is:

$$\frac{104}{0.302} = 345 \text{ amperes per sq.cm.}$$

This density is considerably higher than the value in Table 21, but we have made a rather liberal provision for ventilating ducts

and there is no reason, at this stage, to anticipate that we have exceeded permissible values. However, this must be put to the test at a later stage when we shall have sufficient data to estimate the temperature rise on the basis of the watts lost per square decimeter of peripheral radiating surface at the air-gap.

Since we must provide for 10 conductors (20 component wires) per slot, the height of the winding space must be **at least**

$$10 \times 4.75 = 47.5 \text{ mm.}$$

But it will be impracticable to thread the wires into place with complete avoidance of any lost space. So let us add 5 per cent to the height of the winding space, making it:

$$1.05 \times 47.5 = 50 \text{ mm.}$$

The slot with the wires in place, is drawn in Fig. 74. It is seen from this figure that the total depth is 54 mm. The slot opening is 6 mm. wide.

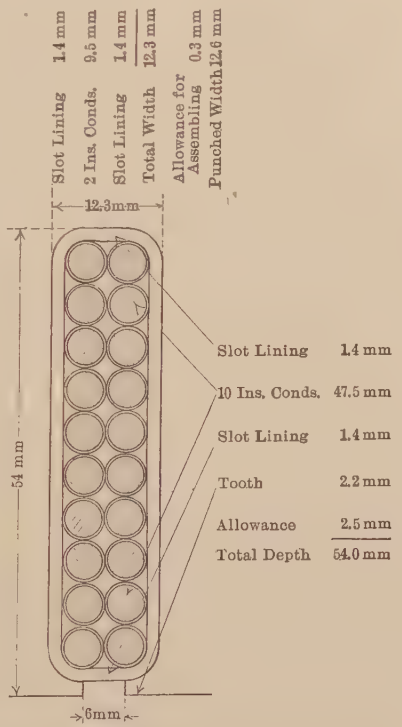


FIG. 74.—Stator Slot of 200 H.P. Motor with Winding in Place.

The Slot Space Factor. The total area of cross-section of copper in the slot amounts to $10 \times 0.302 = 3.02$ sq.cm.

The product of depth and punched width of slot is equal to $1.26 \times 5.40 = 6.80$ sq.cm.

$$\text{Space factor of stator slot} = \frac{3.02}{6.80} = 0.445.$$

It is to be distinctly noted that this slot design is merely a preliminary layout. Should it at a later stage not be found to fulfil the requirements as regards sufficiently-low temperature-

rise at rated load, it will be necessary to consider ways and means of so modifying the design as to fulfil the requirements.

Preliminary Proportions for the Rotor Slot. For reasons which will appear later, there will be a number of rotor slots not differing greatly from the number of stator slots and these rotor slots will be of about the same order of depth as the stator slots, but considerably narrower. The result will be that the rotor tooth density will be fully as low or even lower than the stator tooth density. Let us for the present, consider that the rotor slots are 54 mm. deep and that the crest density in the rotor teeth is, at no load, 15 500 lines per square centimeter.

FIG. 75.—Rotor Slot of 200 H.P. Squirrel-cage Motor.

Let us further assume for the present that the rotor slots are nearly wide open, the shape of a rotor slot being somewhat as indicated in Fig. 75. In the final design, the width of the rotor slot opening may readily be so adjusted as to constitute about 20 per cent of the rotor tooth pitch at the surface of the rotor, the tooth surface thus constituting some 80 per cent of the tooth pitch.

Determination of Cross-section of Air-gap. The stator tooth pitch at the air-gap is:

$$\frac{620\times\pi}{72}=27.1 \text{ mm.}$$

$$\text{Stator slot opening}=6.0 \text{ mm.}$$

The stator tooth surface thus constitutes:

$$\frac{27.1-6.0}{27.1}\times 100=78.0 \text{ per cent}$$

of the stator slot pitch.

Considering the average value of this percentage on both sides of the air-gap, we find it to be:

$$\frac{78.0+80.0}{2}=79.0 \text{ per cent.}$$

If there were no slot openings, the cross-sections of the surfaces from which the lines of each pole emerge and into which they enter, could be taken as $\lambda n \times \tau$.

But the slot openings bring this cross-section down to

$$0.79 \times \lambda n \times \tau.$$

In crossing the air-gap, however, the lines spread out and may be considered to occupy a greater cross-section when half way across. They then gradually converge as they approach the surfaces of the teeth at the other side of the gap. To allow for this spreading, we may increase the cross-section by 15 per cent, bringing it up to $1.15 \times 0.79 \times \lambda n \times \tau$.

For our 200-h.p. induction motor we have:

$$\lambda n = 27.9 \text{ cm.}, \quad \tau = 32.5 \text{ cm.}$$

Cross-section of air-gap $= 1.15 \times 0.79 \times 27.9 \times 32.5 = 825$ sq.cm.

Average air-gap density (at no load) $= \frac{4\,570\,000}{825} = 5550$ lines per sq. cm.

$$\text{Crest density} = 1.7 \times 5550 = 9450.$$

Radial Depth of Air-gap. Let us denote the radial depth of the air-gap in mm. by Δ.

Appropriate preliminary values for Δ are given in Table 24.

TABLE 24.—APPROPRIATE VALUES FOR Δ THE RADIAL DEPTH OF THE AIR-GAP FOR INDUCTION MOTORS.

D, the Air-gap Diameter, (in cm.).	Δ, the Radial Depth of the Air-gap (in mm.), for Various Values of the Peripheral Speed in Meters per Second (mps.).			
	10 mps.	20 mps.	30 mps.	40 mps.
20	0.65	0.75	0.87	1.00
40	0.87	1.05	1.25	1.45
60	1.10	1.35	1.70	1.90
80	1.30	1.7	2.0	2.3
100	1.57	2.0	2.8	2.8
120	1.77	2.3	2.8	3.3

For our 200-h.p. motor, the air-gap diameter is 62 cm. and the peripheral speed is 16.2 mps. Consequently the radial depth of the air-gap is $\Delta = 1.3$ mm.

Preliminary Magnetic Data for Teeth and Air-gap. We have now obtained (or assumed) the densities in the teeth and in the air-gap and we have the lengths of these portions of the magnetic circuit. These data are:

	Length (in cm.) of Portions of Magnet Circuit.	Crest Density at no Load (in Lines per sq.cm.).
Stator teeth.....	5.4	15 500
Rotor teeth.....	5.4	15 500
Air-gap.....	0.13	9 450

The magnetomotive force (mmf.) required to overcome the reluctance of the above-tabulated portions of the magnetic circuit, will, in most designs, constitute a predominatingly-large percentage of the total mmf. As the designer gains experience, he will often be able, after calculating this portion of it, safely to use his judgment in assigning (without detailed calculations), a suitable and relatively-small amount, to provide for the mmf. required for overcoming the reluctance of the stator and rotor cores. In the great majority of cases, this further amount constitutes so small a percentage of the total mmf. per pole, that a very considerable divergence from the value which would be obtained by detailed calculations, would not seriously influence the total. That the length of the magnetic circuit in the stator and rotor cores is much less the greater the number of poles has already been pointed out on page 120 and has been illustrated by the sketches of the 2-, 4-, and 8-pole magnetic circuits in Fig. 62.

Until, however, considerable experience has been gained in calculating the magnetic circuits of induction motors, it is not desirable to trust to obtaining sufficient accuracy by the process of multiplying by a suitable factor the mmf. required for the teeth and the air-gap. It is, however, not necessary to consume time in accurately estimating the mean length of the magnetic path in the stator and rotor cores. On the contrary, it suffices to adopt the assumption that the portions corresponding to one pole may be

taken as equal to the circumferences corresponding to the mean core diameter of the stator and rotor respectively, divided by $2P$, where P is the number of poles. But as a step toward obtaining these values, it is necessary to obtain the external diameter of the stator core discs and the internal diameter of the rotor core discs. These data, in turn, are dependent upon the densities which should be employed behind the slots in the stator and rotor respectively.

Densities in Stator and Rotor Cores. The densities to be employed in the stator and rotor cores are quantities which may be varied between wide limits. In general, however, the core densities should be lower, the greater the value of τ the polar pitch, and $\sim$ the periodicity. For preliminary assumptions, the values given in Table 25 will be found suitable.

TABLE 25.—DENSITIES IN STATOR AND ROTOR CORES.

Periodicity in Cycles per Second.	Values of Polar Pitch (in cm.).	Density in Magnetic Lines per sq.cm.	
		Stator.	Rotor.
25	15	11 500	14 000
	20	10 500	13 500
	30 and larger	10 000	13 000
50	15	10 000	13 000
	20	9 000	12 500
	30 and larger	8 500	12 000

In the present instance we find from the above table that for a periodicity of 25 cycles per second and a polar pitch of 32.5 cm., the stator density should be 10 000 lines per square centimeter, and the rotor density 13 000 lines per square centimeter.

Since the total flux (at no load) is 4.57 megalines per pole, the cross-sections required in the stator and rotor cores are respectively:

$$\text{Cross-section of stator core} = \frac{4\,570\,000}{2 \times 10\,000} = 229 \text{ sq.cm.}$$

$$\text{Cross-section of rotor core} = \frac{4\,570\,000}{2 \times 13\,000} = 176 \text{ sq.cm.}$$

λn is equal to 27.9 cm.

Consequently:

Radial depth of stator punchings (exclusive of slot depth)

$$=\frac{229}{27.9}=8.2 \text{ cm.}$$

Radial depth of rotor punchings (exclusive of slot depth)

$$=\frac{176}{27.9}=6.3 \text{ cm.}$$

External diameter of stator punchings $=62.0+2\times5.4+2\times8.2$
$=62.0+10.8+16.4$
$=89.2$ cm.

Internal diameter of rotor punchings
$=62.0-2\times0.13-2\times5.4-2\times6.3$
$=62.0-0.26-10.8-12.6$
$=38.3$ cm.

Diameter at bottom of stator slots $=62.0+2\times5.4$
$=72.8$ cm.

Diameter at bottom of rotor slots $=62.0-2\times0.13-2\times5.4$
$=50.9$ cm.

Mean diameter of stator core $=\frac{89.2+72.8}{2}=81.0$ cm.

Mean diameter of rotor core $=\frac{38.3+50.9}{2}=44.6$ cm.

Length of sta. mag. circ. per pole $=\frac{81.0\times\pi}{2\times6}=21.2$ cm.

Length of rotor mag. circ. per pole $=\frac{44.6\times\pi}{2\times6}=11.7$ cm.

Compilation of Diameters. It is of interest at this stage to draw up an orderly list of the leading diameters:

External diameter of stator core	892 mm.
Diameter at bottom of stator slots	728 mm.
Internal diameter of stator (D)	620 mm.
External diameter of rotor ($D-2\Delta$)	617.4 mm.
Diameter at bottom of rotor slots	509 mm.
Internal diameter of rotor core	383 mm.

Sketch of Magnetic Portions of Design. We are now in a position to make a preliminary outline drawing of the magnetic parts of our machine. This has been done in Fig. 76 which shows the leading dimensions of the magnetic portions of the design and indicates the locations of the end connections of the stator windings.

Magnetic Reluctance of Sheet Steel. In the design of the induction motor, our magnetic material is exclusively sheet steel. For this material the mmf. data in the first two columns of the table previously given on page 33 will give conservative results.

Tabulated Data of Magnetic Circuit. We now have the lengths of the magnetic paths, the densities, and also data for ascertaining the mmf. required at all parts of the magnetic circuit.

Thus for example:

Density in stator core = 10 000 lines per square centimeter;
Corresponding mmf. from column 2 of table on page 33
= 4.6 ats. per centimeter
Length of magnetic circuit in stator core (per pole) = 21.2 cm.
Mmf. required for stator core = $4.6 \times 21.2 = 98$ ats.

As a further illustration we may give the calculation of the mmf. required for the air-gap:

Crest density in air-gap = 9450 lines per sq.cm.

Corresponding mmf. $= \frac{10}{4\pi} \times 9450 = 0.8 \times 9450 = 7550$ ats. per cm.

Length of magnetic circuit in air-gap = 0.13 cr .
Mmf. required for air-gap = 7550×0.13 = 980 ats.

These illustrations will suffice to render clear the arrangement of the calculations in the following tabulated form:

TABLE 26.—ARRANGEMENT OF MMF. CALCULATIONS.

Part.	(*A*) Length of Mag. Circ. in cm.	Density in Lines per sq.cm.	(*B*) mmf. per cm.	(*A*×*B*) Total mmf.
Stator teeth..	5.4	15 500	22	119
Rotor teeth..	5.4	15 500	22	119
Air-gap.....	0.13	9 450	7550	980
Stator core..	21.2	10 000	4.6	98
Rotor core...	11.7	13 000	9.5	111
		Total mmf. per pole.....		1427 ats.

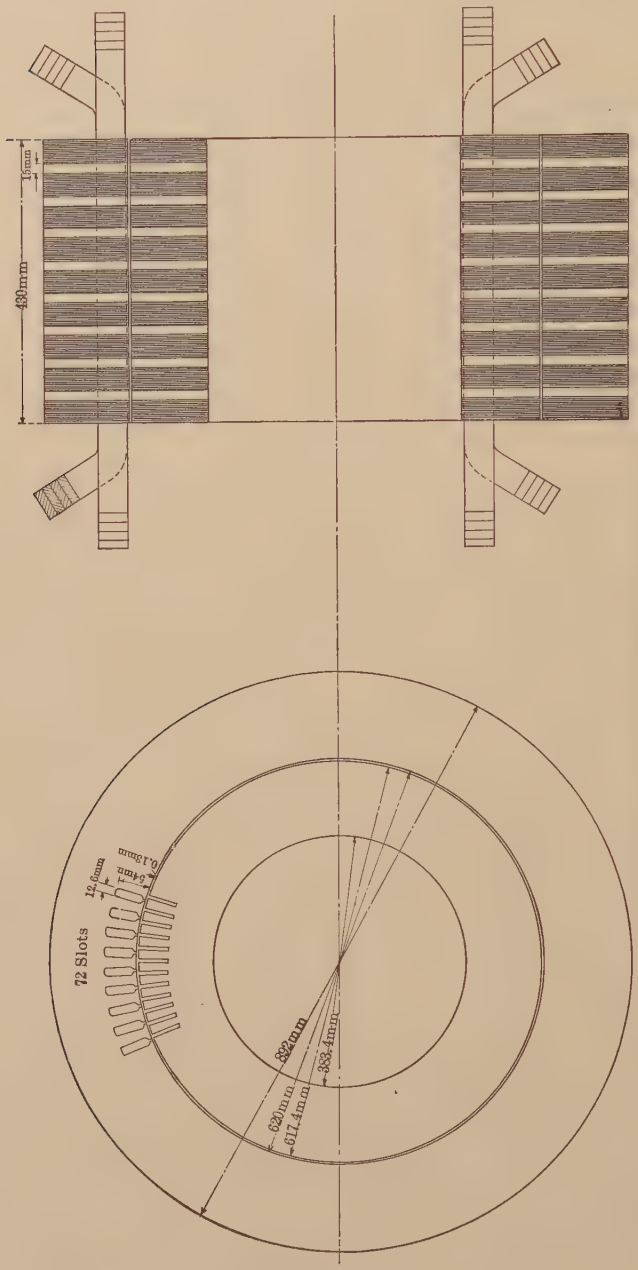


FIG. 76.—Sketch of Magnetic Circuit and Stator Windings of 6-pole, 200 H.P. Squirrel-cage Induction Motor.

In this instance, $\left(\text{since } \frac{980}{1427}\times 100 = 68.5\right)$, the air-gap mmf. is 68.5 per cent of the total required mmf. The mmf. required for air-gap and teeth is:

$$\frac{980+119+119}{1427}\times 100 = \frac{1218}{1427}\times 100 = 85.5 \text{ per cent}$$

of the total required mmf. Attention is drawn to these percentages to bear out the correctness of the assertion on page 134 that considerable inaccuracy in the estimation of the mean length of the magnetic circuit in the stator and rotor cores will not seriously affect the accuracy of the result obtained for the total mmf. per pole. Consequently the use of the rough but time-saving rule to divide by twice the number of poles, the mean periphery of these cores, is shown to be justified.

Resultant mmf. of the Three Phases Equals Twice the mmf. of One Phase. It is a property of the three-phase windings of induction motors that the resultant mmf. of the three phases is twice that exerted by one phase alone. Consequently in our design, each phase must contribute a mmf. of $\frac{1427}{2} = 714$ ats.

In our design, T, the number of turns in series per phase, is equal to 120. The design has 6 poles. Thus we have $\frac{120}{6} = 20$ turns per pole per phase.

Magnetizing Current. The magnetizing current per phase which will suffice to provide the required 714 ats. must obviously amount to:

$$\frac{714}{20} = 35.7 \text{ crest amperes}$$

or

$$\frac{35.7}{\sqrt{2}} = 25.2 \text{ effective amperes.}$$

Since the full-load current is 104 amperes, the magnetizing current is $\frac{25.2}{104}\times 100 = 24.2$ per cent of the full-load current.

No-load Current. The no-load current is made up of two components, the magnetizing current and the current corresponding to the friction and windage loss and the core loss, i.e., to the energy current at no load. It may be stated in advance that the energy current at no load is almost always very small in comparison with the magnetizing current. Since, furthermore, the magnetizing current and the energy current differ from one another in phase by 90 degrees, it follows that their resultant, the no-load current, will not differ in magnitude appreciably from the magnetizing current. The calculation of the energy current is thus a matter of detail which can well be deferred to another stage. But to emphasize the relations of the quantities involved, let us assume that the friction, windage and core loss of this motor will later be ascertained to be a matter of some 4500 watts. This is 1500 watts per phase. The pressure per phase is $\left(\frac{1000}{\sqrt{3}}=\right)577$ volts. Consequently the energy component of the current consumed by the motor at no load is $\frac{1500}{577}=2.6$ amperes. The no-load current thus amounts to $\sqrt{25.2^2+2.6^2}=25.3$ amperes. In other words the no-load current and the magnetizing current differ from one another in magnitude by less than one-half of one per cent, in this instance. Although of but slight practical importance, it may be interesting to show that they differ quite appreciably in phase. For we have for the angle of phase difference between the no-load current and the magnetizing current:

$$\tan^{-1}\frac{2.60}{25.2}=\tan^{-1}0.103=5.9°.$$

Thus the current in this motor when it is running unloaded, lags $(90-5.9=)$ 84.1° behind the pressure; in other words, its power-factor is equal to $(\cos 84.1°=)$ 0.104.

γ, the Ratio of the No-load Current to the Full-load Current. It is convenient to designate by γ, the ratio of the no-load current to the full-load current. For our motor we have:

$$\gamma=\frac{25.2}{104}=0.242.$$

Practical use will be made of this ratio at a later stage in the design of this motor.

H, **the Average Number of Slots per Pole.** It is also convenient to adopt a symbol for the average of the number of slots per pole on the stator and rotor. We have definitely determined upon the use of 12 slots per pole on the stator. We have reserved to a later stage of the calculations the determination of the **precise** number of rotor slots. However it may here be stated that it is preferable to employ a number of rotor slots not widely differing from the number of stator slots. Thus as a preliminary assumption we may take 12 as the average of the numbers of slots per pole on stator and rotor. Designating this quantity by *H* we have;

$$H = 12.$$

THE CIRCLE RATIO

We have now all the necessary data for determining a quantity for which we shall employ the symbol σ and which, for reasons which we shall come to understand as we proceed, we shall term the "circle ratio." This quantity is of great utility to the practical designer. Although σ cannot be predetermined with any approach to accuracy, it so greatly assists one's mental conceptions from the **qualitative** standpoint as to make ample amends for its **quantitative** uncertainty.

We have seen that at no load, the current consumed by an induction motor lags nearly 90 degrees behind the pressure. Let us picture to ourselves a motor with no friction or core loss and with windings of no resistance. In such a motor the no-load current would be exclusively magnetizing and would lag 90 degrees behind the pressure.

Let us assume a case where, at no load, the current is 10 amperes. The entire magnetic flux emanating from the stator windings will cross the zone occupied by the secondary conductors (i.e., the conductors on the rotor) and pass down into the rotor core. If the circumstance of the presence of load on the motor were not to disturb the course followed by the magnetic lines, then the magnetizing component of the current flowing into the motor

would remain the same with load as it is at no load. If the motor were for 100 volts per phase, then, in this imaginary case where the flux remains undisturbed as the load comes on, we could calculate in a very simple way the current flowing into the motor for any given load. To illustrate; let us assume that a load of 3000 watts is carried by this hypothetical motor. A load of 3000 watts corresponds to an output of 1000 watts per phase. Assuming a motor with no internal losses, the **input** will also amount to 1000 watts per phase. Since the pressure per phase is 100 volts, the energy component of the current input per phase is $\left(\frac{1000}{100}=\right)10$ amperes. Since the magnetizing component is 10 amperes the resultant current per phase is $\sqrt{10^2+10^2}=14.1$ amperes. The vector diagram corresponding to these conditions is given in Fig. 77. The resultant current lags behind the terminal pressure by

$$\tan^{-1}\frac{10}{10}=\tan^{-1}1.0=45°.$$

The power-factor is:

$$\cos 45°=0.707.$$

Let us double the load. The energy component of the current increases to $2\times10=20$ amperes, as shown in Fig. 78.

The total current increases to

$$\sqrt{10^2+20^2}=22.4 \text{ amperes.}$$

The angle of lag becomes:

$$\tan^{-1}\frac{10}{20}=\tan^{-1}0.5=26.6°.$$

The power-factor increases to:

$$\cos 26.6°=0.894.$$

Let us again double the load, thereby increasing the energy component of the current to 40 amperes, and the angle of lag to 14.0° as shown in Fig. 79. The total current is now 41.3 amperes and the power-factor is:

$$\left(\frac{40.0}{41.3} =\right) 0.97.$$

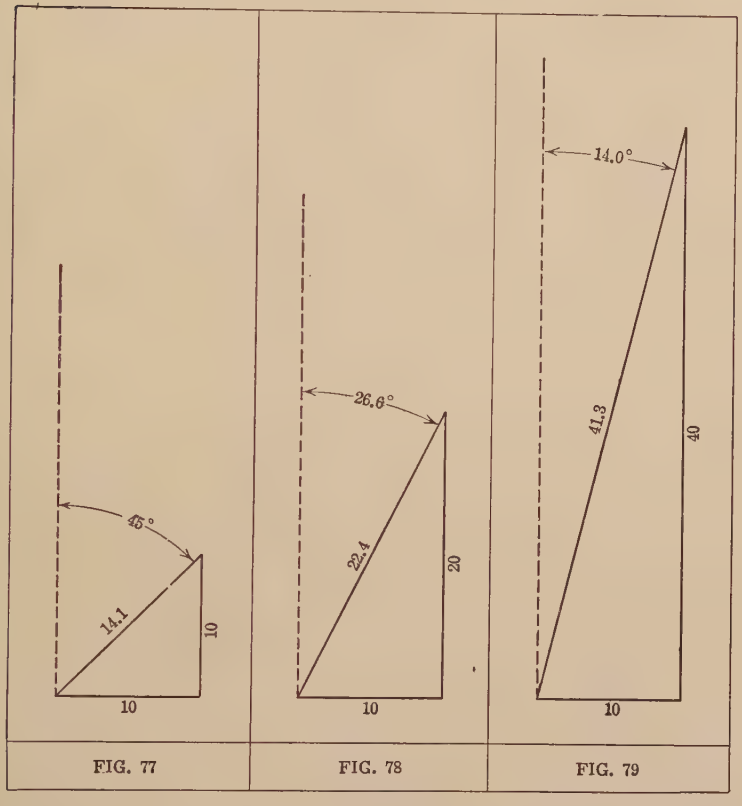


FIG. 77 FIG. 78 FIG. 79

FIGS. 77 to 79.—Vector Diagrams Relating to a Hypothetical Polyphase Induction Motor without Magnetic Leakage.

At this rate we should quickly approach unity power-factor. The curve of increase of power-factor with load, would be that drawn in Fig. 80. It is to be especially noted that in the diagrams in Figs. 77, 78 and 79, the vertical ordinates indicate the energy

components of the total current and the horizontal ordinates indicate the wattless (or magnetizing) components of the current.

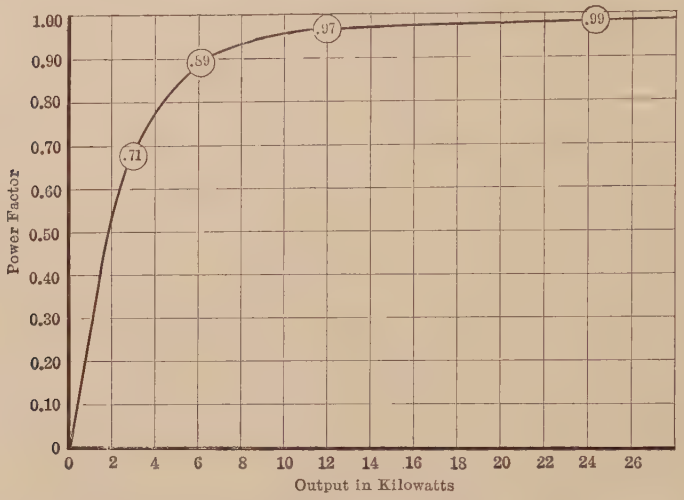


FIG. 80.—Curve of Power-factor of Hypothetical Polpyhase Induction Motor without Magnetic Leakage.

It would be very nice if we could obtain the conditions indicated in the diagrams of Figs. 77, 78 and 79. That is to say, it would be very nice if the magnetizing component of the current remained

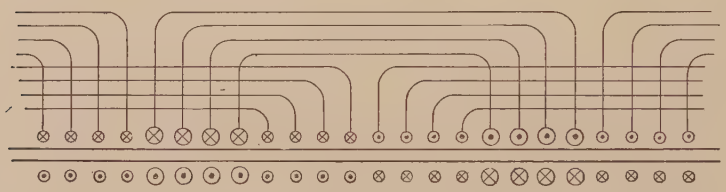


FIG. 81.—Diagrammatic Representation of the Distribution of the Magnetomotive Forces in the Stator and Rotor Windings of a Three-phase Induction Motor.

constant with increasing load. But this is not the case. As the load increases, the current in the rotor conductors (which was

negligible at no load), increases. The combined effect of the current in the stator and rotor conductors is to divert a portion of the flux out of the path which it followed at no load. In Fig. 81 are indicated diagramatically a few slots of the stator and rotor windings. Considering the 12 left-hand conductors, the current is indicated as flowing (at the moment) down into the plane of the paper in the stator conductors and up out of the plane of the paper in the rotor conductors. This has the same effect (as regards the resultant mmf. of the stator and rotor conductors), as would be occasioned by the arrangement indicated in Fig. 82, in which the stator and rotor conductors constitute a single spiral. Obviously the mmf. of this spiral would drive the flux along the air-gap between the stator and rotor surfaces. The reluctance of the circuit traversed by the magnetic flux thus increases gradually (with increasing current input), from the relatively low reluctance of the main magnetic circuit traversed by the entire flux at no load, up to the far higher reluctance of the circuit traversed by practically the entire magnetic flux with the rotor at standstill, the pressure at the terminals of the stator windings being maintained constant throughout this entire range of conditions. Consequently the magnetizing component of the current consumed by the motor increases as the load increases. Thus instead of the diagrams in Figs. 77, 78 and 79, we should have the three diagrams shown at the right hand in Fig. 83. The corresponding diagrams at the left hand in Fig. 83 are simply those of Figs. 77, 78 and 79 introduced into Fig. 83 for comparison. In both cases, the no-load current is 10 amperes. But in the practical case with magnetic leakage, the loads calling respectively for energy components of 10, 20 and 40 amperes (loads of 3000, 6000 and

FIG. 82.—Diagram Indicating a Solenoidal Source of mmf. Occasioning a Flux Along the Air-gap, Equivalent to the Leakage Flux in an Induction Motor.

12 000 watts) involve magnetizing components of 10.6, 12.1 and 18.2 amperes.

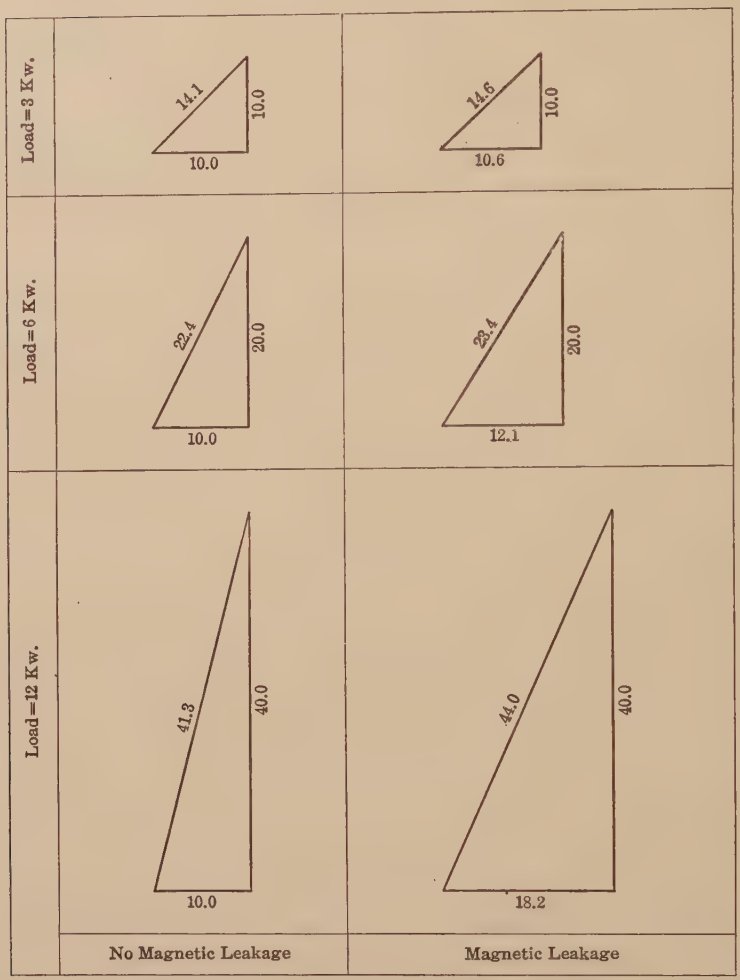


FIG. 83.—Vector Diagrams for Hypothetical Motor *without* Magnetic Leakage (at Left), and for Actual Motor *with* Magnetic Leakage (at Right).

The total current inputs are increased as follows:

Load (in Watts).	Current Input per Phase.	
	On Assumption of no Magnetic Leakage.	On Assumption of Magnetic Leakage.
0	10.0	10.0
3 000	14.1	14.6
6 000	22.4	23.4
12 000	41.3	44.0

The power-factors for the two cases are:

Load in Watts.	Power-factor.	
	No Magnetic Leakage.	Magnetic Leakage.
0	0	0
3 000	0.707	$\left(\frac{10.0}{14.6}=\right)$ 0.685
6 000	0.894	$\left(\frac{20.0}{23.4}=\right)$ 0.855
12 000	0.970	$\left(\frac{40.0}{44.0}=\right)$ 0.910

The curve of power-factor for the case with magnetic leakage is given in Fig. 84, and there has been reproduced dotted in this

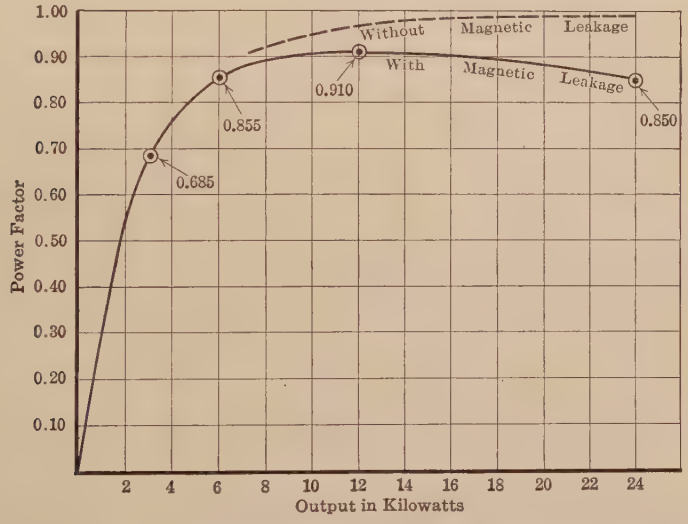


FIG. 84.—Curves of Power-factor of Hypothetical Induction Motor *without* Magnetic Leakage (Dotted Line) and of Actual Motor *with* Magnetic Leakage (Full Line).

figure the power-factor curve for the case with no magnetic leakage which has already been given in Fig. 80.

In Fig. 85, the hypothenuses of the right-hand diagrams of Fig. 83 have been superposed, and their right-hand extremities are seen to lie upon the circumference of a semi-circle with a diameter of 200 amperes. In Fig. 85 the magnetizing current of 10 amperes is denoted by AB. The diameter of the semi-circle is BD. We have:

$$AB = 10;$$
$$BD = 200.$$

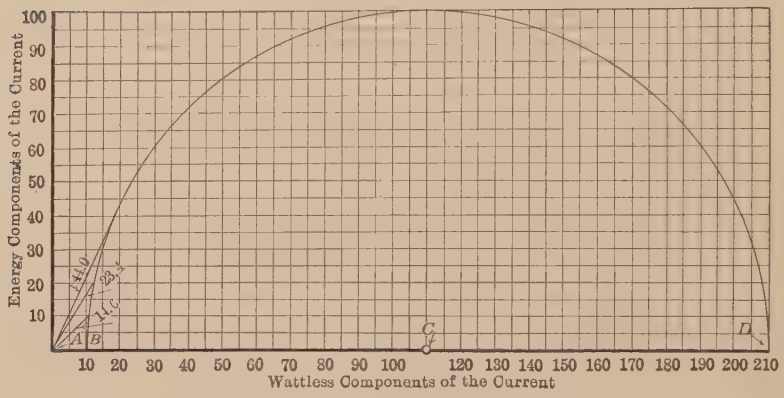


FIG. 85.—Circle Diagram for a Polyphase Induction Motor with a No-load Current of 10 Amperes and a Circle-ratio of 0.050.

The quantity which we termed the "circle ratio" and which we designated by the symbol σ, is the ratio of AB to BD. For this case we have:

$$\frac{AB}{BD} = \frac{10}{200} = 0.050;$$

$$\sigma = 0.050.$$

The semi-circle in Fig. 85 is the locus of the extremities of the vectors representing the current flowing into the stator winding. If, for any value of the current input, we wish to ascertain its phase relations, we draw an arc with A as a center and with the value of the current as a radius. The intersection of this arc with the semi-circle, constitutes one extremity of the vector

representing the current, and A constitutes its other extremity. The horizontal projection of this vector is its wattless component and the vertical projection is its energy component. Consequently we also have the convenient relation that the vertical component is, for constant pressure, a direct measure of the power absorbed by the motor. We shall give further attention to these important relations at a later stage.

The circle ratio is a function of λg, τ, Δ and H. Knowing λg, σ, Δ and H we can obtain a rough value for σ for any motor. In other words, having selected these four quantities for a motor which we are designing, we can obtain σ. Having calculated AB, the magnetizing current, by the methods already set forth on page 139, we may divide it by σ and thus obtain BD. For we have:

$$BD = \frac{AB}{\sigma}.$$

We are now in a position to construct, for the 200 h.p. motor which we are designing, a diagram of the kind represented in Fig. 85.

We must first determine σ. We have:

$$\lambda g = 43.0 \text{ cm.};$$
$$\tau = 32.5 \text{ cm.};$$
$$\Delta = 1.3 \text{ mm.};$$
$$H = 12.$$

Knowing these four quantities, the "circle factor," σ, may be obtained from Table 27. For our motor we find from the table,

$$\sigma = 0.041.$$

The values of σ in Table 27 apply to designs with intermediate proportions as regards slot openings. Should both stator and rotor slots be very nearly closed (say 1 mm. openings), the value of σ would be increased by—say—20 per cent or more. On the other hand, were both stator and rotor slots wide open, σ would be decreased by—say—some 20 per cent below the values set forth in the table. It cannot be too strongly emphasized that we cannot predetermine σ at all closely. We can, however, take 0.041 as a probable value for σ in the case of our design. If, on test, the observed value were found to be within 10 per cent of 0.041,

TABLE 27.—VALUES OF THE CIRCLE RATIO, σ, FOR VARIOUS VALUES OF H, λg, τ AND Δ, FOR SQUIRREL-CAGE MOTORS.

	Values of σ for:—										
	$H=6$					$H=8$					
	Δ	$\tau=20$	$\tau=25$	$\tau=30$	$\tau=35$	Δ	$\tau=20$	$\tau=25$	$\tau=30$	$\tau=35$	$\tau=40$
$\lambda g=10$	0.8	0.108	0.096	0.091	0.082	0.8	0.092	0.080	0.073	0.071	0.070
	1.2	0.118	0.104	0.096	0.094	1.2	0.100	0.088	0.080	0.078	0.076
	1.6	0.128	0.112	0.104	0.100	1.6	0.109	0.094	0.086	0.082	0.080
	2.0	0.138	0.121	0.112	0.110	2.0	0.118	0.100	0.093	0.091	0.088
	2.4	0.148	0.132	0.120	0.120	2.4	0.124	0.105	0.098	0.097	0.094
$\lambda g=15$	0.8	0.096	0.088	0.081	0.075	0.8	0.083	0.073	0.067	0.064	0.062
	1.2	0.108	0.096	0.088	0.082	1.2	0.090	0.080	0.074	0.070	0.067
	1.6	0.114	0.104	0.096	0.088	1.6	0.100	0.086	0.080	0.076	0.074
	2.0	0.128	0.114	0.104	0.100	2.0	0.108	0.095	0.088	0.083	0.081
	2.4	0.136	0.120	0.112	0.112	2.4	0.114	0.100	0.094	0.090	0.088
$\lambda g=20$	0.8	0.085	0.080	0.075	0.067	0.8	0.075	0.066	0.061	0.058	0.056
	1.2	0.096	0.088	0.081	0.076	1.2	0.082	0.073	0.067	0.064	0.062
	1.6	0.104	0.093	0.088	0.081	1.6	0.090	0.080	0.074	0.070	0.067
	2.0	0.118	0.104	0.096	0.092	2.0	0.098	0.088	0.081	0.077	0.075
	2.4	0.124	0.112	0.104	0.101	2.4	0.107	0.095	0.088	0.085	0.081
$\lambda g=30$	0.8	0.071	0.070	0.064	0.058	0.8	0.062	0.056	0.052	0.048	0.046
	1.2	0.080	0.076	0.067	0.064	1.2	0.070	0.064	0.059	0.055	0.052
	1.6	0.088	0.084	0.076	0.070	1.6	0.077	0.071	0.065	0.060	0.058
	2.0	0.100	0.090	0.084	0.080	2.0	0.086	0.079	0.072	0.068	0.066
	2.4	0.110	0.100	0.094	0.088	2.4	0.094	0.086	0.079	0.075	0.072
$\lambda g=50$	0.8	0.053	0.052	0.048	0.044	0.8	0.043	0.042	0.040	0.038	0.036
	1.2	0.061	0.060	0.056	0.048	1.2	0.052	0.049	0.046	0.044	0.042
	1.6	0.068	0.066	0.060	0.054	1.6	0.062	0.060	0.052	0.050	0.048
	2.0	0.080	0.077	0.068	0.064	2.0	0.070	0.067	0.059	0.059	0.056
	2.4	0.088	0.084	0.076	0.072	2.4	0.080	0.074	0.067	0.064	0.062

TABLE 27 (*Continued*).—VALUES OF THE CIRCLE RATIO, σ, FOR VARIOUS VALUES OF H, λg, τ AND Δ FOR SQUIRREL-CAGE MOTORS.

	Values of σ for:																
	$H=10$						$H=15$						$H=20$				
	Δ	$\tau=20$	$\tau=25$	$\tau=30$	$\tau=35$	$\tau=40$	Δ	$\tau=20$	$\tau=25$	$\tau=30$	$\tau=35$	$\tau=40$	Δ	$\tau=25$	$\tau=30$	$\sigma=35$	$\tau=40$
$\lambda g=10$	0.8	0.075	0.072	0.064	0.058	0.056	0.8	0.056	0.050	0.046	0.046	0.045	0.8	0.040	0.038	0.037	0.035
	1.2	0.086	0.078	0.070	0.066	0.062	1.2	0.063	0.057	0.053	0.051	0.048	1.2	0.046	0.044	0.042	0.040
	1.6	0.095	0.084	0.074	0.072	0.065	1.6	0.070	0.065	0.058	0.057	0.052	1.6	0.053	0.049	0.046	0.044
	2.0	....	0.090	0.080	0.076	0.070	2.0	0.078	0.072	0.066	0.062	0.058	2.0	0.061	0.057	0.053	0.049
	2.4	....	0.096	0.084	0.080	0.074	2.4	0.085	0.081	0.072	0.067	0.060	2.4	0.068	0.063	0.058	0.053
$\lambda g=15$	8.8	0.069	0.066	0.059	0.052	0.051	0.8	0.050	0.046	0.043	0.042	0.040	0.8	0.037	0.035	0.034	0.033
	1.2	0.079	0.072	0.064	0.060	0.055	1.2	0.058	0.054	0.050	0.048	0.046	1.2	0.043	0.041	0.039	0.038
	1.6	0.086	0.078	0.069	0.066	0.060	1.6	0.066	0.061	0.055	0.054	0.050	1.6	0.051	0.047	0.044	0.042
	2.0	0.098	0.086	0.075	0.071	0.064	2.0	0.074	0.069	0.062	0.059	0.055	2.0	0.057	0.054	0.051	0.047
	2.4	....	0.090	0.080	0.076	0.068	2.4	0.081	0.076	0.070	0.065	0.059	2.4	0.066	0.061	0.056	0.051
$\lambda g=20$	0.8	0.064	0.061	0.055	0.049	0.046	0.8	0.047	0.042	0.038	0.040	0.038	0.8	0.035	0.034	0.032	0.030
	1.2	0.073	0.068	0.060	0.055	0.050	1.2	0.054	0.050	0.046	0.044	0.042	1.2	0.042	0.040	0.038	0.035
	1.6	0.082	0.074	0.066	0.062	0.055	1.6	0.062	0.058	0.052	0.050	0.048	1.6	0.049	0.046	0.042	0.040
	2.0	0.090	0.080	0.071	0.067	0.059	2.0	0.070	0.066	0.059	0.056	0.052	2.0	0.056	0.054	0.049	0.045
	2.4	0.097	0.086	0.077	0.072	0.064	2.4	0.077	0.073	0.066	0.062	0.057	2.4	0.060	0.061	0.055	0.050
$\lambda g=30$	0.8	0.053	0.052	0.047	0.042	0.039	0.8	0.041	0.038	0.036	0.034	0.033	0.8	0.033	0.030	0.028	0.026
	1.2	0.063	0.059	0.053	0.049	0.044	1.2	0.049	0.046	0.042	0.039	0.037	1.2	0.038	0.036	0.034	0.032
	1.6	0.073	0.066	0.059	0.055	0.048	1.6	0.056	0.052	0.048	0.046	0.043	1.6	0.044	0.042	0.040	0.039
	2.0	0.081	0.072	0.066	0.060	0.053	2.0	0.064	0.059	0.055	0.051	0.048	2.0	0.051	0.049	0.047	0.043
	2.4	0.089	0.079	0.071	0.066	0.058	2.4	0.072	0.067	0.061	0.057	0.053	2.4	0.059	0.056	0.052	0.048
$\lambda g=50$	0.8	0.035	0.040	0.040	0.034	0.034	0.8	0.035	0.031	0.028	0.027	0.025	0.8	0.028	0.026	0.023	0.021
	1.2	0.048	0.047	0.046	0.039	0.037	1.2	0.043	0.039	0.036	0.034	0.031	1.2	0.034	0.032	0.030	0.027
	1.6	0.059	0.054	0.051	0.044	0.042	1.6	0.051	0.046	0.043	0.041	0.038	1.6	0.041	0.038	0.036	0.035
	2.0	0.066	0.060	0.057	0.050	0.047	2.0	0.058	0.054	0.050	0.047	0.043	2.0	0.047	0.045	0.043	0.039
	2.4	0.074	0.067	0.060	0.055	0.053	2.4	0.066	0.059	0.054	0.051	0.049	2.4	0.055	0.052	0.048	0.044

the result should be considered to be as close as could reasonably be expected.

This indication of our inability to closely predetermine σ should not lead to a disparagement of its utility. Most practical purposes are amply satisfied, in commercial designing, when we can, in our preliminary work, construct the circle diagram with this degree of accuracy. It will be seen in the course of this treatise that the efficiency and power-factor can be closely predetermined in spite of this degree of indeterminateness in the circle ratio.

At the author's request, Mr. F. H. Kierstead has recently analyzed the test results of 130 polyphase induction motors, and from these results he has derived a formula for estimating the circle ratio. The range of dimensions of the 130 motors may be seen from the following:

Δ	varies from	0.64	mm.	to 2.54	mm.
λg	"	10	cm.	to 61	cm.
τ	"	11	cm.	to 84	cm.
H	"	7		to 32	
D	"	20	cm.	to 310	cm.

Fifty-eight of the motors were of American manufacture and the remaining 72 were of British, German, Swedish, Swiss, French, and Belgian manufacture. Our object was to obtain a formula which would yield an approximately representative value, regardless of the detail peculiarities of design inherent to the independent views and methods of individual designers and manufacturers.

For squirrel-cage motors Kierstead's formula is as follows:

$$\sigma = C\left(\frac{0.20}{\lambda g} + \frac{0.48}{\tau} + \frac{3.0}{H^2}\right).$$

C is a function of Δ, and may be obtained from Table 28.

TABLE 28.—VALUES OF C IN KIERSTEAD'S FORMULA FOR THE CIRCLE RATIO.

Δ (in mm.).	C.	Δ (in mm.).	C.
0.60	0.71	1.60	1.25
0.80	0.88	1.80	1.31
1.00	0.99	2.00	1.39
1.20	1.09	2.20	1.45
1.40	1.17	2.40	1.52

Of course it is realized that the width and depth of the slot, width of the slot openings, the arrangement of the end connections, the degree of saturation, and other factors which have not been taken into consideration, necessarily materially modify the value of σ, the circle ratio. Indeed, all experienced designers know that the introduction of some extreme proportion with respect to dimensions additional to those taken into account in Kierstead's formula, will be accompanied by quite a large increase or decrease in the circle ratio. Consequently, the formula must be taken as corresponding to average proportions. When any extreme departure is made from these average proportions, it will be expedient for the designer to employ his judgment as to the extent by which he should modify the value of the circle ratio as obtained from the formula.

The investigation is at present being continued with a view to arriving at a term to be introduced into the formula to take into account the slot dimensions. In its present form, however, Kierstead's formula constitutes a valuable aid to the designer of induction motors. Table 27 was derived before Kierstead undertook his investigation and is based upon data of 71 out of the 130 motors analyzed by Kierstead.

Below is given a bibliography of a number of useful articles relating to the estimation of the circle-ratio.

I. Chapter IV (p. 29) of Behrend's "Induction Motor."
II. "The Magnetic Dispersion in Induction Motors," by Dr. Hans Behn-Eschenburg; Journal Inst. Elec. Engrs., Vol. 32, (1904), pp. 239 to 294.
III. Chapter XXI on p. 470 of the 2d Edition of Hobart's "Electric Motors."
IV. "The Leakage Reactance of Induction Motors," by A. S. McAllister, Elec. World for Jan. 26, 1907.
V. "The Design of Induction Motors," by Prof. Comfort A. Adams, Transactions Amer. Inst. Elec. Engrs., Vol. 24 (1905), pp. 649 to 687.
VI. "The Leakage Factor of Induction Motors," by H. Baker and J. T. Irwin, Journal Inst. Elec. Engrs., Vol. 38, (1907) pp. 190 to 208.

γ and σ the Two Most Characteristic Properties of a Design. We have now determined the two most important characteristics of our design. These are:

γ, the ratio of the no-load current to the full-load current;
σ, the circle ratio.

For our 200-h.p. design we have:

$$\gamma = 0.242;$$
$$\sigma = 0.041.$$

The Circle Diagram of the 6-pole 200-H.P. Motor. For the full-load current we have:

$$I = 104 \text{ amperes.}$$

$$\text{No-load current} = \gamma \times I = 0.242 \times 104 = 25.2 \text{ amperes.}$$

This value of 25.2 amperes is plotted as AB in Fig. 86.

$$\text{Diameter of circle} = \frac{AB}{\sigma} = \frac{25.2}{0.041} = 615 \text{ amperes.}$$

In Fig. 86, the diameter of the circle is made equal to 615 amperes.

Significance also attaches to the length AD which, in Fig. 86 equals (25.2+615=) 640 amp. AD represents a quantity

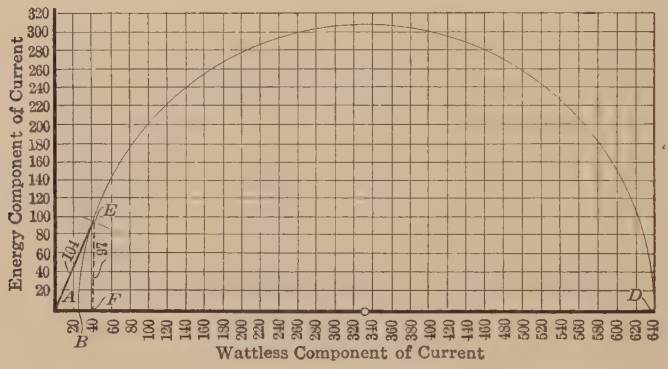


FIG. 86.—The Circle Diagram of the 200-H.P. Motor.

which may be termed the "ideal short-circuit current." It is that current which, if the stator and rotor windings were of zero resistance and if there were no core loss, would be absorbed by each phase of the stator windings if the normal pressure of 1000 volts (577 volts per phase) were maintained at the terminals of the motor, the periodicity being maintained at 25 cycles per second. In other words, the reactance, S, of the motor, under these conditions, is:

$$S = \frac{577}{640} = 0.902 \text{ ohm per phase.}$$

Estimation of the Full-load Power-factor. It will be remembered that up to this point we have employed for the full-load power-factor, a preliminary value taken from Table 18. This preliminary value was $G=0.91$.

In Fig. 86 the line AE has been drawn with a length of 104 amperes, from A to the point E of intersection with the circle. EF, the vertical projection of this line, is found to be 97 amperes.

Thus the diagramatically-obtained value of the power-factor is

$$G=\frac{97}{104}=0.932.$$

Let us take it as 0.93 and let us revise our data in accordance with this new value. We have:

Full-load efficiency (η) = 0.91 (original assumption);

Full-load power-factor (G) = 0.93 (revised value);

Full-load current (I) $= \frac{0.91}{0.93}\times 104 = 102$ amperes;

No-load current $=25.2$ amperes;

$$\lambda=\frac{25.2}{102}=0.247 \text{ (revised value)};$$

$\sigma=0.041$ (unchanged).

The diagram, drawn to a larger scale, and with the slight modification of employing this new value of 102 amperes for I (the full-load current), is drawn in Fig. 87. It is seen that the vertical projection of I is now 95. This gives us for the power-factor:

$$G=\frac{95}{102}=0.93$$

confirming the readjusted value.

The Stator I^2R Loss. We shall estimate the stator I^2R loss by first estimating the mean length of one turn of the winding, and from this obtaining the total length of conductor per phase. From this length and the already-adopted cross-section of the

conductor, the resistance per phase is obtained. We may denote the resistance per phase by R. The stator I^2R loss at rated load is equal to $3I^2R$.

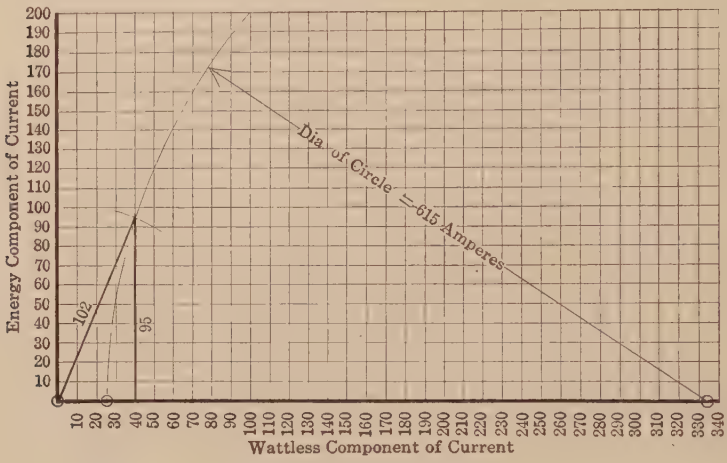


FIG. 87.—Revised Circle Diagram of 200 H.P. Motor.

The **mean length of turn** (mlt.) may be roughly estimated from the formula:

$$\text{mlt.} = 2\lambda g + K\tau.$$

K is a function of the pressure for which the stator is wound, and may be taken from Table 29.

TABLE 29.—VALUES OF K IN FORMULA FOR MLT.

Terminal Pressure for which Stator is Wound.	K.
500 (or less)	2.5
1 000	3.0
2 000	3.5
4 000	4.0
6 000	4.5
8 000	5.0
10 000	5.5
12 000	6.0

For our 1000-volt 200 h.p. design, we find the value:

$$K = 3.0.$$

Therefore:

$$\text{mlt.} = 2 \times 43.0 + 3.0 \times 32.5 = 183.5 \text{ cm.}$$

For the number of stator turns in series per phase we have:

$$T = 120$$

Consequently the length of conductor per phase equals:

$$120 \times 183.5 = 22\,000 \text{ cm.}$$

The cross-section of the conductor has already been fixed at:

$$0.302 \text{ sq.cm.}$$

The I^2R loss at 60° Cent. is desired. At this temperature, the specific resistance of commercial copper wire may be taken at 0.00000200 ohm per cm. cube.

Thus at 60° Cent. the resistance of each phase of the stator winding is:

$$R = \frac{22\,000 \times 0.0000020}{0.302} = 0.146 \text{ ohm.}$$

For the stator I^2R loss at full load we have:

$$3 \times 102^2 \times 0.146 = 4540 \text{ watts.}$$

Stator IR Drop at Full Load. Instead of a flux corresponding to $\left(\frac{1000}{\sqrt{3}} = \right)$ 577 volts per phase, we shall, at full load, have a lesser flux. It will, in fact, correspond to:

$$577 - 102 \times 0.146 = 577 - 15 = 562 \text{ volts.}$$

This is an internal drop of:

$$\frac{15}{577}\times 100 = 2.6 \text{ per cent.}$$

Strictly speaking, we ought, therefore, to take into account the decreased magnetic densities with increasing load. Cases arise where it would be of importance to do this, but in the present instance such a refinement would be devoid of practical interest and will not be undertaken.

THE DETERMINATION OF THE CORE LOSS.

For the stator core, the best low-loss sheet-steel should be employed notwithstanding that its cost is still rather high. The advantage in improved performance will much outweigh the very slight increase which its use occasions in the Total Works Cost. In the rotor, the reversals of magnetism are, during normal running, at so low a rate that the rotor core loss is of but slight moment. It is consequently legitimate to employ a cheaper grade of material in the construction of the rotor cores. But in practice it is usually more economical to use the same grade as for the stator cores notwithstanding the absence of need for the better quality. By the time the outlay for waste and the outlay for wages and for general expenses are added, there will be but trifling difference in the cost of the two qualities.

The data given in Table 30 are well on the safe side. Individual designers will ascertain by experience in how far they can rely upon obtaining better material.

TABLE 30.—DATA FOR ESTIMATING THE CORE LOSS IN INDUCTION MOTORS.

Density in Stator Core in Lines per Square Centimeter.	Core Loss in Stator Core in Watts per kg. for Various Periodicities.		
	$\sim=15$	$\sim=25$	$\sim=50$
6 000	1.1	2.2	5.0
8 000	1.7	3.0	7.4
10 000	2.2	4.0	10.0
12 000	2.6	5.2	
14 000	3.2	6.2	

For our 25-cycle motor, we have employed in the stator core, a density of 10 000 lines per sq.cm. and we shall consequently estimate the core loss on the basis of 4.0 watts per kg. of total weight of stator core.

Estimation of Weight of Stator Core

External diameter of stator core = 89.2 cm.
Internal diameter of stator core = 62.0 cm.

The gross area of a stator core plate, i.e., the area before deducting the area of the slots, is equal to:

$$\frac{\pi}{4}(89.2^2 - 62.0^2) = 3260 \text{ sq.cm.}$$

Area of 72 stator slots $= 72 \times 5.4 \times 1.26 = 490$ sq.cm.

Net area of stator core plate $= (3260 - 490 =)$ 2770 sq.cm .

The volume of the stator core is obtained by multiplying this area by λn, i.e., by 27.9.

Volume of sheet steel in stator core $= 2770 \times 27.9 = 77\,400$ cu.cm.
Weight of 1 cu.cm. of sheet steel = 7.8 grams.

Therefore:

$$\text{Weight of sheet steel in stator core} = \frac{77\,400 \times 7.8}{1000} = 603 \text{ kg.}$$

The accuracy with which core losses can be estimated is not such as to justify dealing separately with the teeth and the main body of the stator core. It suffices simply to multiply the net weight in kg. by the loss in watts per kg. corresponding to the density in the main body of the stator core. Therefore:

$$\text{Stator core loss} = 603 \times 4.0 = 2410 \text{ watts.}$$

Core Loss in Rotor. The periodicity of reversal of magnetization in the rotor core is so low that there should not be much core loss in the rotor. But to allow for minor phenomena and to keep on the safe side, it is a good rule to assess the rotor core

loss at 10 per cent of the stator core loss. For our machine we have:

$$\text{Rotor core loss} = 2410 \times 0.10 = 240 \text{ watts.}$$

Input to Motor and to Rotor at Rated Load. We cannot yet check our preliminary assumption of an efficiency of 91 per cent at rated load. On the basis of this efficiency, the input to the motor at its rated load is:

$$\frac{200 \times 746}{0.91} = 164\,000 \text{ watts.}$$

The losses in the stator amount to a total of:

$$4540 + 2410 = 6950 \text{ watts.}$$

Deducting the stator losses at full load from the input to the motor at full load we ascertain that:

$$164\,000 - 6950 = 157\,050 \text{ watts}$$

are transmitted to the rotor.

A Motor is a Transformer of Energy. A motor receives energy in the form known as "electricity." An account can be rendered of all the energy received. In the case we are considering, the full-load input is at the rate of 164 000 watts. Energy flows into the motor at the rate of 164 kw. hr. per hour.* In certain instances it is more convenient to make some equivalent statement with other units of power and time. Thus we may say that energy flows into the motor at the rate of 164 000 watt seconds per second. The amount of energy corresponding to the expenditure of one watt for one second is termed by the physicist, one joule.

$$1 \text{ joule} = 1 \text{ watt second.}$$

It requires 4190 joules to raise the temperature of 1 kg. of water by 1° cent. Thus we have:

$$1 \text{ kg. calorie (kg.cal.)} = 4190 \text{ joules.}$$

* The proposal to designate as 1 kelvin the amount of energy corresponding to 1 kw. hr., is gradually gaining favor amongst European engineers.

The energy expended in lifting a weight of 1 kg. through a height of 1 meter is equivalent to 9.81 joules. In other words 1 kg.m. = 9.81 joules.

The energy need not necessarily be expended in **lifting** a weight. If a force of 1 kg. is exerted through a distance of 1 meter, an amount of energy equal to 9.81 joules is expended in the process.

The amount of energy corresponding to the expenditure of 1 h.p. for 1 sec. is equal to 746 joules; or

1 h.p. sec. = 746 joules.

It is convenient to bring together these equivalents:

1 watt second = 1 joule;
1 h.p. second = 746 joules;
1 kg.m. = 9.81 joules;
1 kg.cal. = 4190 joules.
1 kelvin = 3 600 000 joules.

The rated output of our motor may be expressed as:

1. 200 h.p.;
1*a*. 200 h.p. sec. per sec.;
2. 149 200 watts;
2*a*. 149 200 joules per second.
3. $\left(\frac{149\,200}{9.81} = \right)$ 15 200 kg.m. per second.
4. $\left(\frac{149\,200}{4190} = \right)$ 35.6 kg.cal. per second.

Let us concentrate our attention on Designation 3 for the rated output. In accordance with this designation, the motor's output at its rated load is

15 200 kg.m. per sec.

If the shaft of our motor is supplied with a gear wheel of 1 meter radius through which it transmits the energy to another engaging gear wheel, and thence to the driven machinery, then

for every revolution of the armature, the distance travelled by a point on the periphery of the gear wheel is

$$2\times\pi=6.28 \text{ meters.}$$

At the motor's synchronous speed of 500 r.p.m., the peripheral speed of the gear wheel is

$$6.28\times\frac{500}{60}=52.3 \text{ meters per sec.}$$

(Although such a high speed would not, in practice, be employed, it has been preferable, for the purpose of the present discussion, to consider a gear wheel of 1 meter radius.)

Since at full load the motor's output is:

$$15\ 200 \text{ kg.m. per sec.}$$

and since the peripheral speed of the gear wheel is 52.3 meters per sec., it follows that the pressure at the point of contact between the driving and the driven gear teeth is

$$\frac{15\ 200}{52.3}=290 \text{ kg.}$$

This is the force exerted at a radius of 1 meter. We say that at full load the motor exerts a " torque " of

$$290\times1=290 \text{ kg. at 1 meter leverage.}$$

Had the radius of the gear been only 0.5 m. instead of 1.0 meter, then the force would have been $2\times290=580$ kg., but the " torque " would still have been equivalent to

$$0.5\times580=290 \text{ kg. at 1 meter leverage.}$$

In dealing with torque it is usually convenient to reduce it to terms of the force in kg. at 1 meter leverage, irrespective of the actual leverage of the point of application of the force.

At rated load our motor will **not** run at the synchronous speed of 500 r.p.m., but at some slightly lower speed. If the "slip" is 1 per cent, then the full-load speed will be

$$0.99 \times 500 = 495 \text{ r.p.m.}$$

If the "slip" is 2 per cent, then the full-load speed will be

$$0.98 \times 500 = 490 \text{ r.p.m.}$$

Since the rated output will nevertheless be 200 h.p. in these two cases, the torque will be; **not** 290 kg. at 1 m. leverage, but

$$\left(\frac{290}{0.99} =\right) 293 \text{ kg. for the case of 1 per cent slip}$$

and

$$\left(\frac{290}{0.98} =\right) 296 \text{ kg. for the case of 2 per cent slip.}$$

The torque (i.e., the force at 1 m. leverage) is only one component of the energy delivered. The other component is the distance traversed by a hypothetical point at 1 meter radius revolving at the angular speed of the rotor. For 2 per cent slip, the distance traversed in 1 sec. by such a hypothetical point, is

$$0.98 \times 52.4 = 51.3 \text{ meters.}$$

The energy delivered from the motor in each second is thus

$$296 \times 51.3 = 15\,200 \text{ kg.m.}$$

The power (or rate of deliverance of energy) is

$$15\,200 \text{ kg.m. per sec.;}$$

or,

$$9.81 \times 15\,200 = 149\,200 \text{ watts;}$$

or,

$$\frac{149\,200}{746} = 200 \text{ h.p.}$$

The torque exerted by the rotor conductors is greater than that finally available at the gear teeth. The discrepancy corresponds to the amount of the I^2R loss in the rotor conductors and to the amount of the rotor core loss and the windage and bearing friction. The **rotor core loss** and the **friction** come in just the same category as an equal amount of external load. Thus if 3 h.p. is required to supply the rotor core loss plus friction, then the **output from the rotor conductors** is 203 h.p. as against the ultimate output of 200 h.p. **from the motor.**

But the I^2R loss in the rotor conductors comes in an altogether different category. The loss can only come about as the result of a cutting of the flux across the rotor conductors. In other words, the rotor conductors must not travel quite as fast as the revolving magnetic field. Consequently the rotor will run at a speed slightly less than synchronous; there will be a "slip" between the revolving magnetic field and the revolving rotor. It is only in virtue of such slip that the rotor conductors can be the seat of any force. Thus the torque is inseparably associated with the "slip" and the "slip" will be greater the greater the load. As the "slip" and torque increase, the rotor I^2R loss also increases and the speed of the rotor decreases.

If the I^2R loss in the rotor conductors amounts to 1 per cent of the input to the rotor, then the "slip" will be 1 per cent. If the I^2R loss is increased to 2 per cent, then the "slip" increases to 2 per cent. If, finally, the I^2R loss amounts to 100 per cent of the input to the rotor, then the "slip" will be 100 per cent, i.e., the motor will be at rest, but it may nevertheless be exerting torque. For such a condition it is desirable to regard matters from the following standpoint. If the rotor is suitably secured so that it cannot rotate; then if electricity is sent into the motor a certain portion will be transmitted by induction to the rotor circuit, just as if it constituted the secondary circuit of a transformer. The input to the rotor will under these conditions consist of the I^2R loss in its conductors and the core loss. There is no other outlet for the energy sent into the rotor and it all becomes transformed into energy in the form of heat in the rotor conductors and in the rotor core. Since under such conditions the rotor core loss is negligible in comparison with the rotor I^2R loss, we may regard the input to the rotor as practically

identical with the rotor I^2R loss. The rotor I^2R loss is 100 per cent of the input to the rotor and the slip is 100 per cent.

The Locus of the Rotor Current in the Circle Diagram. Let us consider the hypothetical case of a rotor with a number of conductors equal to the number of stator conductors. We can describe this as an arrangement with a 1:1 ratio of transformation. Any actual induction motor can be considered to have its equivalent with a 1:1 ratio, and it is convenient and usual to investigate certain properties of induction motors by considering the equivalent rotor winding with a 1:1 ratio.

The vector diagram of the stator and rotor currents is shown in Fig. 88 for our 200-h.p. motor with a 1:1-ratio rotor. The diagram is drawn for full load and consequently the stator current, AE, is equal to 102 amperes. The rotor current, AF must have such direction and magnitude that the resultant, AB, shall be equal to the no-load magnetizing current, which we have already found to be 25.2 amperes. AF is consequently equal, as regards phase and magnitude, to EB and is found graphically to amount to 96 amperes. In practice, it is more convenient to represent the rotor current by lines drawn from B as the origin and connecting B to the points where the corresponding primary vectors intersect the circumference of the semi-circle. In Fig. 89 are drawn the stator and rotor vectors for two values of the stator current, namely 80 amperes and 300 amperes. The corresponding values of the rotor current are 72 and 286 amperes.

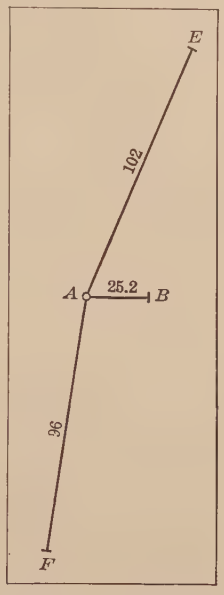


FIG. 88.—Vector Diagram Indicating the Primary and Secondary Currents in the 200-H.P. Induction Motor at its Rated Load.

The Rotor I^2R Loss of the 200-H.P. Motor at Its Rated Load. It is still desirable to postpone to a later stage the design of the rotor conductors. But let us assume that the full-load slip will be 2.0 per cent. Then the rotor I^2R loss at full load will be 2.0 per cent of the input to the rotor. The input to the rotor

has been ascertained (on page 160) to be 157 050 watts. Thus the rotor I^2R loss at full load is $0.02 \times 157\,050 = 3140$ watts.

We have now determined (or assumed) all the full-load losses except windage and bearing friction.

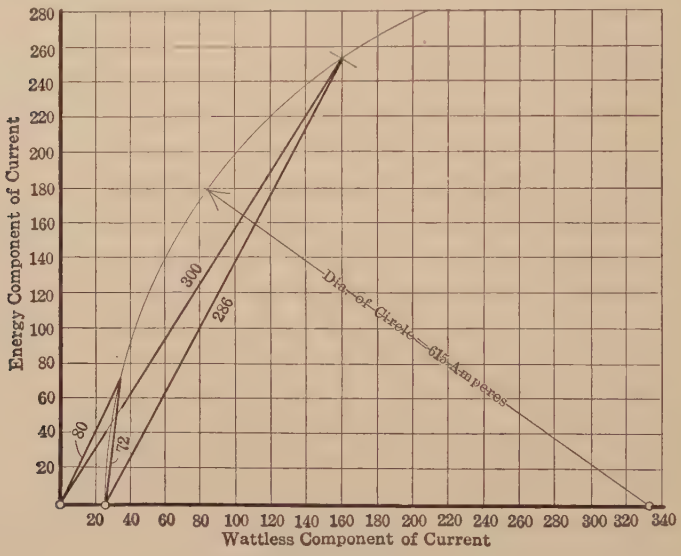


FIG. 89.—Diagram Indicating Stator and Rotor Current Vectors.

Friction Losses. It is very hard to generalize as regards friction losses. A reasonable estimate may, however, be made from Table 31.

TABLE 31.—DATA FOR ESTIMATING THE FRICTION LOSS IN BEARINGS AND WINDAGE IN INDUCTION MOTORS.

$D^2\lambda g$ (D and λg in dm.)	Total Friction Loss in Watts, for Various Rated Speeds.		
	400 r.p.m.	800 r.p.m.	1600 r.p.m.
10	150	340	1200
20	280	620	2000
30	370	800	2500
40	460	1000	3100
50	550	1250	3700
100	800	1750	5600
200	1100	2400	8000

For our motor we have:

$$D^2\lambda g = 6.2^2 \times 4.3$$
$$= 166.$$

From Table 31 we ascertain by interpolation that the friction loss may be taken as 1300 watts.

Output from Rotor Conductors. The output from the rotor conductors is made up of the 200-h.p. output from the motor, the friction, and the rotor core loss. The two latter amount to

$$1300 + 240 = 1540 \text{ watts } \left(= \frac{1540}{746}\right) = 2.06 \text{ h.p.}$$

Thus we may take the output from the rotor conductors as

$$200 + 2.06 = 202 \text{ h.p.}$$

Referring back to page 163 we find that for 2 per cent slip and 200 h.p., we require 296 kg. at 1 meter leverage.

Consequently the torque required to be exerted by the rotor conductors is:

$$\frac{202}{200} \times 296 = 299 \text{ kg. at 1 meter leverage.}$$

In other words, the full-load torque exerted by the rotor conductors is 299 kg.

The Torque Factor. At full load, the input to the rotor is 157 050 watts. Thus the input to the rotor in watts per kg. of torque developed, which we may term the "torque factor," is:

$$\frac{157\,050}{299} = 525.$$

This factor will be useful to us in studying the starting torque.

The "Equivalent" Resistance of the Rotor. We have on p. 157 made an estimation of the stator resistance and have ascertained it to be 0.146 ohm per phase at 60° cent.

The stator I^2R loss at full load is $3 \times 102^2 \times 0.146 = 4540$ watts.

The rotor I^2R loss at full load is 3140 watts.

We shall employ a squirrel-cage rotor (for which we shall soon design the conductors) and we may consider its " equivalent " resistance to be:

$$\frac{3140}{4540} \times 0.146 = 0.101 \text{ ohm.}$$

Without introducing any serious inaccuracy we may (when the motor is running in the neighborhood of synchronous speed), ascertain the rotor I^2R loss for any value of the stator current (except for very small loads) by multiplying this " equivalent " resistance by three times the square of the stator current.

Rotor at Rest. But when the rotor is at rest, the currents circulating in its windings are of the line periodicity and the conductors have an apparent resistance materially greater than their true resistance. Attention was called to a related phenomenon in a paper presented by A. B. Field, in June, 1905, before the American Institute of Electrical Engineers and entitled " Eddy Currents in Large Slot-Wound Conductors."* Recently the application of the principle has been incorporated in the design of squirrel-cage induction motors to endow them with desired values of starting torque. The multiplier by which the apparent resistance may be obtained from the true resistance may be found approximately from the formula:

$$\text{Multiplier} = 0.15 \times (\text{depth of rotor bar in cm.}) \times \sqrt{\text{periodicity.}}$$

In our case we have:

$$\text{Multiplier} = 0.15 \times 5.4 \times \sqrt{25} = 4.05.$$

This multiplier only relates to the embedded portions of the conductors. The portions of the length where the conductors cross the ventilating ducts are not affected, nor are the end rings subject to this phenomenon. A rough allowance for this can be made by reducing the multiplier to $0.8 \times 4.05 = 3.24$.

Thus at standstill the " apparent " resistance of our rotor will be $3.24 \times 0.101 = 0.327$ ohm.

* Vol. 24, p. 761.

The "Equivalent" Resistance of the Motor. At standstill, the motor, regarded as a whole, and without distinction between primary and secondary may be considered as having a resistance of (0.146+0.327=) 0.473 ohm per phase.

On page 154 we have seen that the reactance of the motor is:

$$S=0.902 \text{ ohm per phase.}$$

For the impedance of the motor at standstill we have:

$$\sqrt{0.473^2+0.902^2}=1.03 \text{ ohm per phase.}$$

THE STARTING TORQUE

In starting this motor we should not put it at once across the full pressure of 1000 volts but should apply only one-half or one-third of this pressure. Let us examine the conditions at standstill when half pressure is applied.

We shall have $\frac{500}{\sqrt{3}}=288$ volts per phase.

The stator current will be:

$$\frac{288}{1.03}=280 \text{ amperes per phase.}$$

For the rotor I^2R loss we have $3\times280^2\times0.327=77\,000$ watts.

The torque developed is obtained by dividing this value by the torque factor, i.e., by 525.

$$\text{Torque}=\frac{77\,000}{525}=146 \text{ kg. at 1 meter leverage.}$$

The full-load torque is 296 kg.

Consequently with half the normal pressure at the motor, we shall obtain

$$\frac{146}{296}=49.5 \text{ per cent of full-load torque.}$$

We obtain the half pressure at the motor by tapping off from the middle point of a starting compensator. The connections (for a quarter-phase motor), are as shown in Fig. 90. With this arrangement, the current drawn from the line will be only half of the current taken by the motor. Since the motor takes 280 amperes, the current from the line is only

$$\left(\frac{280}{2} =\right) 140 \text{ amperes.}$$

Thus with but $\left(\frac{140}{102} =\right)$ 1.37 times full-load current from the line, we can start the motor with 50 per cent of full-load torque.

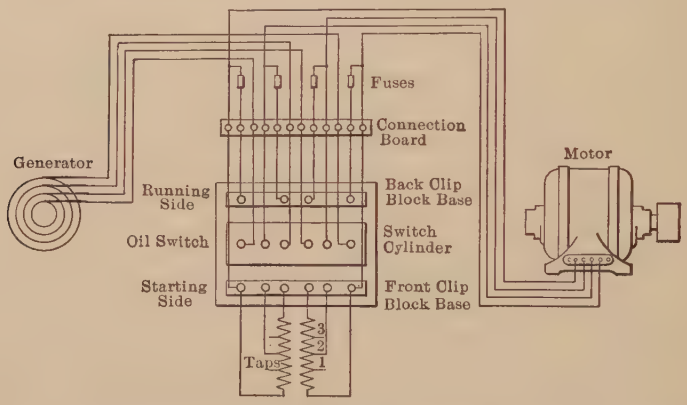


FIG. 90.—Connections for Starting Up an Induction Motor by Means of a Compensator (sometimes called an auto-transformer).

This excellent result is achieved by employing deep rotor conductors and does not involve the necessity of resorting to high slip during normal running.

Circle Diagram for These Starting Conditions. For half pressure, the magnetizing current will, strictly speaking, be a little less than half its former value of 25.2 amperes, since the magnetic parts are worked at lower saturation. But for simplicity let us take it at $\left(\frac{25.2}{2} =\right)$ 12.6 amperes. The circle ratio, σ, varies a little with the saturation, but let us take it at its former

value of 0.041. We can now construct the circle diagram shown in Fig. 91, in which:

$$AB = 12.6 \text{ amperes};$$

$$BD = \frac{12.6}{0.041} = 308 \text{ amperes};$$

$$AD = 12.6 + 308 = 321;$$

$$AE = 280 \text{ amperes}.$$

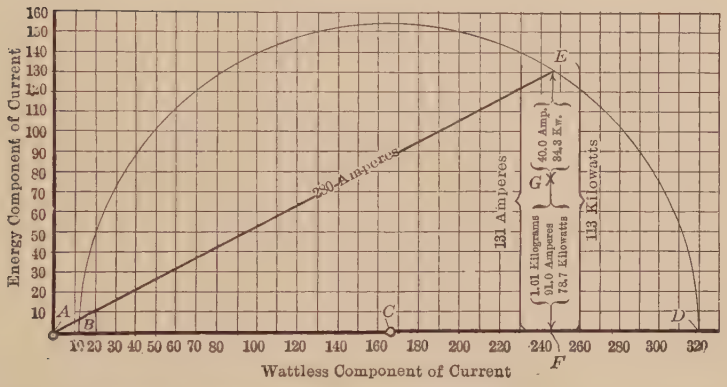


FIG. 91.—Starting Torque Diagram for 200-H.P. Motor when Connected for Starting from Half the Normal Line Pressure.

A vertical line EF is drawn from E to the base line. This is the vertical projection of 280 amperes and represents the energy component of the input to the motor under these conditions of standstill. From the diagram we ascertain graphically that:

$$EF = 131 \text{ amperes}.$$

Consequently the input to the motor is:

$$3 \times 288 \times 131 = 113\,000 \text{ watts}.$$

The stator I^2R loss $= 3 \times 280^2 \times 0.146 = 34\,300$ watts.

Subtracting this from the input, we ought to obtain the I^2R loss in the rotor.

$$\text{Rotor } I^2R \text{ loss} = 113\,000 - 34\,300 = 78\,700 \text{ watts}.$$

This is in good agreement with the value of 77 000 watts which we obtained for the rotor I^2R loss by applying the analytical method.

EF is, for constant pressure, a measure of the input to the motor and is to the scale of

$$\frac{113\,000}{131} = 865 \text{ watts per ampere.}$$

[It can also be seen that this would be the case from the circumstance that at a pressure of 288 volts per phase, the input is equal to:

$$(3 \times 288 \times I) = (865\ I) \text{ watts}].$$

The rotor I^2R loss of 78 700 watts may then be represented by the height *FG* corresponding to $\left(\frac{78\,700}{865} = \right) 91.0$ amperes. and the stator I^2R loss may be represented by the remainder, *GE*, corresponding to

$$\left(\frac{34\,300}{865} = \right) 40.0 \text{ amperes.}$$

It also necessarily follows that *FG* (and corresponding vertical heights for other conditions similarly worked out), is a measure of the torque. When used in this way, the scale is

$$\frac{146}{91.0} = 1.60 \text{ kg. (at 1 meter leverage) per amp.}$$

Some General Observations Regarding the Circle Diagram. It is this adaptability to the forming of mental pictures of the occurrences, which renders the circle diagram of great importance in the design of induction motors. All the various calculations involved in induction-motor design may be carried through by analytical methods but it is believed that these **exclusively** analytical methods are inferior in that they disclose no simple picture of the occurrences. It is well known that in practice the locus of the extremity of the stator-current vectors is rarely more than

a very crude approximation to the arc of a circle and that the designer can only consider as rough approximations the results he deduces on the basis of the circle assumption. But when employed with judgment the plan is of great assistance and suffices to yield reasonable results.

THE SQUIRREL CAGE

Let us now design our rotor's squirrel cage. As yet we have merely prescribed that the full-load I^2R loss in the squirrel cage shall be 3140 watts, that the slots shall be 54 mm. deep and that we shall employ a number of slots not differing very materially from 72, the number of stator slots.

The Number of Rotor Slots. Were we to employ 72 rotor slots, the motor would have a strong " cogging " tendency. That is to say, if, with the rotor at rest, pressure were applied to the stator terminals, there would be a strong tendency for the rotor to " lock," at a position in which there would be a rotor slot directly opposite to each stator slot. This tendency would very markedly interfere with the development of the starting torque calculated in a preceding section. The choice of 71 or 73 slots would eliminate this defect, but might lead to an unbalanced pull, slightly decreasing the radial depth of the air-gap at one point of the periphery and correspondingly increasing its depth at the diametrically opposite point. This excentricity once established, the gap at one side would offer less magnetic reluctance than the gap diametrically opposite, thus increasing the dead-point tendency. But by selecting 70 or 74 slots, this defect is also eliminated. Let us determine upon 70 rotor slots for our design.

It may in general be stated that the tendency to dead points at starting will be less.

1. The smaller the greatest common divisor of the numbers of stator and rotor slots.
2. The greater the average number of stator and rotor slots per pole.
3. The less the width of the slot openings.
4. The greater the resistance of the squirrel-cage.
5. The deeper the rotor slots.

The influence of the last two factors will be better understood if it is pointed out that they determine the rotor I^2R loss at starting, and we have already seen that the starting torque is proportional to the I^2R loss in the rotor. The fluctuations in the starting torque arising from variations in the relative positions of the stator and rotor slots, will obviously be a **smaller percentage** of the average starting torque, the greater the **absolute value** of the average starting torque. Thus if the average starting torque is very low, a small fluctuation might periodically reduce it to zero; i.e., the motor would have dead points. If, on the other hand, the average starting torque is high, these same fluctuations, superposed on this high average starting torque, would still leave a high value for the minimum torque, and there would be no dead points.

The Pitch of the Rotor Slot. The external diameter of the rotor is $(620-2\times1.3=)$ 617.4 mm.

The diameter at the bottom of the slots is:

$$(617.4-2\times54=)\ 509.4 \text{ mm.}$$

Consequently the rotor slot pitch at the bottom of the slot is:

$$\frac{509.4\times\pi}{70}=22.8 \text{ mm.}$$

The depth of the rotor conductor can be practically identical with the depth of the slot. Therefore depth of rotor conductor $=54$ mm.

Ratio of Transformation. We have 72 stator slots and 10 conductors per slot; hence a total of 720 stator conductors; as against only 70 rotor conductors. The ratio of transformation is thus:

$$720 : 70=10.3 : 1.$$

We have already estimated that for a 1 : 1 ratio, the rotor current would, at full load, amount to 96 amperes. We are now able to state that with the actual ratio of 10.3 : 1, the current in the rotor face conductors will be $10.3\times96=990$ amperes.

At each end of the rotor core, the rotor conductors will terminate in end rings. It will be desirable, for structural reasons, to

space these end rings (in this particular motor), 2.0 cm. away from the ends of the laminated core. Consequently the length of each conductor between end rings will amount to

$$43+2\times2=47 \text{ cm.}$$

The I^2R loss in the 70 rotor conductors will be equal to that in a single conductor of the same (as yet undetermined) cross-section, but with a length of

$$70\times47=3290 \text{ cm.}$$

We have seen that we wish, at full load, to have a loss of 3140 watts in the squirrel cage. Were the loss in the end rings negligible (the entire 3140 watts being dissipated in the 70 face conductors, then their section would be so chosen that they should have an aggregate resistance of:

$$\frac{3140}{990^2}=0.00320 \text{ ohm.}$$

Since at 60° cent., the specific resistance of commercial copper is 0.00000200, the required cross-section would be:

$$\frac{3290\times0.00000200}{0.00320}=2.06 \text{ sq.cm.}$$

Since the depth of a rotor conductor is 54 mm., its width would thus require to be:

$$\frac{206}{54}=3.82 \text{ mm.}$$

But we cannot afford to provide so much material in the end rings as to render them of practically negligible resistance. Let us plan to allow a loss of 628 watts (20 per cent) in the end rings, the remaining $(3140-628=)$ 2512 watts occurring in the slot conductors. This will require an increase in the width of the slot conductors, to:

$$\frac{3140}{2512}\times3.82=4.78 \text{ mm.} \quad \text{or} \quad 4.8 \text{ mm.}$$

Thus the slot conductors will have a cross-section of 54 mm.×4.8 mm. The rotor slot will also be 54 mm. deep and 4.8 mm. wide, and there will be no insulation on the rotor conductors.

The End Rings. The object in minimizing the loss in the end rings will have been divined. Since we want to have the apparent resistance at starting, as great as possible for a given "slip" at normal load, we want to concentrate the largest practicable portion of the loss in the slot conductors since it is these conductors which manifest the phenomenon of having, for high-periodicity currents, a loss greatly in excess of that occurring when they are traversed by currents of the low periodicity corresponding to the "slip."

The Current in the End Rings. It can be shown* that the current in each end ring is equal to:

$$\frac{\text{Number of rotor conductors}}{\pi \times \text{number of poles}} \times \text{current per slot conductor.}$$

For our motor we have:

$$\text{Full-load current in each end ring} = \frac{70}{\pi \times 6} \times 990 = 3660 \text{ amperes.}$$

Since we have allowed 628 watts for the loss in the end rings, we have a loss of 314 watts per end ring, and we have:

$$\text{Resistance for one end ring} = \frac{314}{3660^2} = 0.0000234 \text{ ohm.}$$

Each end ring will have a mean diameter of:

$$\frac{61.7 + 50.9}{2} = 56.3 \text{ cm.}$$

and a mean circumference of $56.3\pi = 177$ cm.

Consequently the cross-section of each end ring is:

$$\frac{177 \times 0.00000200}{0.0000234} = 15.1 \text{ sq.cm.}$$

* The proof is given in Chapter XXIII (pp. 490 to 492) of the 2d Edition of the author's "Electric Motors" (Whittaker & Co., 1910).

Let us make the cross-section up with a height of 54 mm. and a width of 28 mm. A conductor of this cross-section will, at

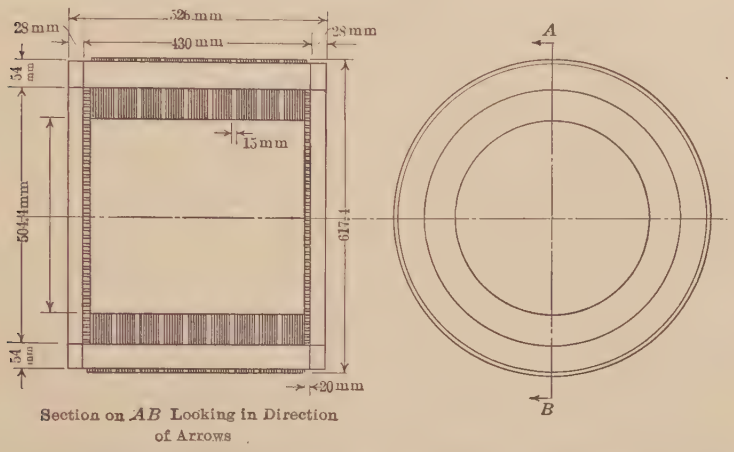


FIG. 92.—Outline of Squirrel-cage Rotor for 200-H.P. Polyphase Induction Motor.

25 cycles, be subject to a slight increase in resistance, due to ordinary skin effect, but this will not be of sufficient amount to add much to the starting torque. The increase in resistance will in fact amount to about 10 per cent.

A sketch of the rotor is given in Fig. 92, and a section of a slot and two teeth, in Fig. 93. Since the slot pitch at the bottom of the slot is 22.8 mm. and since the slot is 4.8 mm. wide, we have a tooth width of $22.8-4.8=18.0$ mm.

With $\left(\frac{70}{6}=\right)$ 11.7 teeth per pole, the cross-section of the magnetic circuit at the bottom of the rotor teeth is

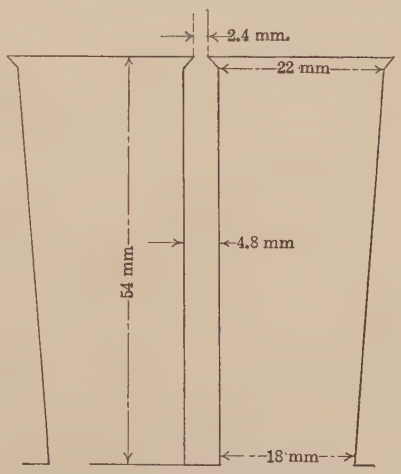


FIG. 93.—Rotor Slot and Teeth for 200-H.P. Squirrel-cage Motor.

$$11.7\times1.8\times27.9=588 \text{ sq.cm.}$$

The crest density at the bottom of the rotor teeth is:

$$\frac{4\,570\,000}{588} \times 1.7 = 13\,200 \text{ lines per sq.cm.}$$

THE EFFICIENCY

We have now estimated all the losses in our 200-h.p. motor. Let us bring them together in an orderly table:

At full load we have:

Stator I^2R loss	4 540 watts
Stator core loss	2 410 "
Rotor I^2R loss	3 140 "
Rotor core loss	240 "
Friction and windage loss	1 300 "
Total of all losses	11 630 watts
Output at full load	149 200 "
Input at full load	160 830 watts

$$\text{Full-load efficiency} = \frac{149\,200}{160\,830} \times 100 = 93.0 \text{ per cent.}$$

Our original assumption for the full-load efficiency was 91.0 per cent. Consequently, strictly speaking, we ought to revise several quantities, such as current input, stator I^2R loss, rotor I^2R loss, and ultimately obtain a still closer approximation to the efficiency. But the object of working through this example has been to convey information with respect to the *methods of carrying out* the calculations involved in the design of an induction motor; and there is no special reason to undertake the above mentioned revision.

THE HALF-LOAD EFFICIENCY

At half load, the energy component of the full-load current input, will, sufficiently exactly for our purpose, be halved. This energy component is:

$$G \times I = 0.93 \times 102 = 95 \text{ amperes.}$$

At half load, the wattless component will be slightly in excess of 25.2 amperes, the no-load current. Let us take it as 29 amperes. The total current input at half load will be:

$$\sqrt{\left(\frac{95}{2}\right)^2+29^2}=55.5 \text{ amperes.}$$

The stator I^2R loss at half load will be:

$$\left(\frac{55.5}{102}\right)^2\times 4540=1340 \text{ watts.}$$

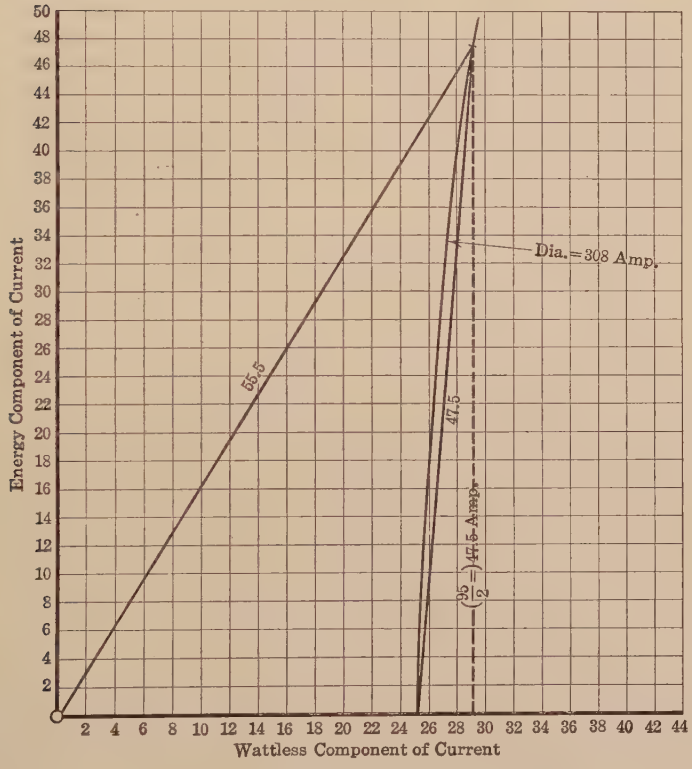


FIG. 94.—Diagram for Obtaining Rotor Current at Half-load.

The readiest way to obtain the corresponding value of the secondary current is to make the circle-diagram construction indicated in Fig. 94. From this construction we see that for a 1 : 1 ratio, the secondary current would be 47.5 amperes. We

have seen on page 165 that at full load, when the 1 : 1 secondary current was 96 amperes, the rotor I^2R loss was 3140 watts.

Consequently at half load, the rotor I^2R loss will be:

$$\left(\frac{47.5}{96}\right)^2 \times 3140 = 770 \text{ watts.}$$

Thus at half load we have:

Stator I^2R loss	1 340 watts
Stator core loss	2 410 "
Rotor I^2R loss	770 "
Rotor core loss	240 "
Friction and windage loss	1 300 "
Total of all losses	6 060 watts
Output at half load	74 600 "
Input at half load	80 660 watts

$$\text{Half-load efficiency} = \frac{74\,600}{80\,660} \times 100 = 92.5 \text{ per cent.}$$

Making similar calculations for other loads we obtain the following inclusive table of the losses at various loads:

TABULATION OF LOSSES AND EFFICIENCIES AT 60° CENT.

Percentage of rated output	0	25	50	75	100	125
Amperes input per phase	25.2	35.0	55.5	77.0	102	127
Stator I^2R loss	280	550	1340	2580	4540	7100
Stator core loss	2410	2410	2410	2410	2410	2410
Rotor I^2R loss	20	300	770	1750	3140	4900
Rotor core loss	240	240	240	240	240	240
Friction and windage loss	1300	1300	1300	1300	1300	1300
Total of all losses	4250	4800	6060	8280	11630	15950
Output in watts	0	37400	74600	112000	149200	186600
Output in horse-power	0	50	100	150	200	250
Input	4250	42200	80660	120280	160830	202550
η	0	0.885	0.925	0.930	0.930	0.921

These efficiencies are plotted in the curve of Fig. 95.

Estimation of the Temperature Rise. In making a rough estimate of the heating of the motor the first step consists in calculating the "watts per square decimeter (sq.dm.) of radiating surface at the air-gap."

The "radiating surface at the air-gap" is taken as the surface over the ends of the stator windings. While this surface will vary considerably according to the type of winding employed, a representative basis may be obtained from the formula:

$$\text{"Equiv. radia. surface at air-gap"} = \pi \times D \times (\lambda g + k\tau).$$

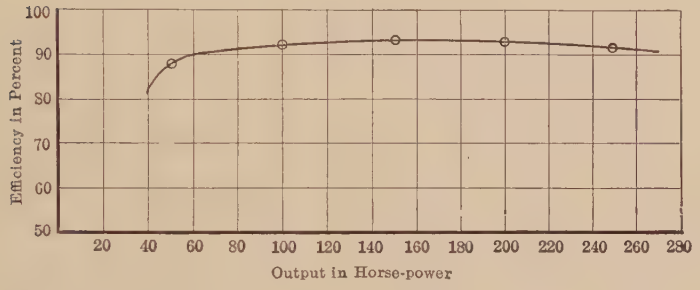


FIG. 95.—Efficiency Curve for 200-H.P. Polyphase Induction Motor.

The k in this formula is a factor which is a function both of the normal pressure for which the motor is built, and of the polar pitch, τ. Suitable values will be found in Table 32.

TABLE 32.—DATA FOR ESTIMATING THE RADIATING SURFACE.

Rated Pressure of the Motor.	Values of k in the Formula: Equiv. rad. sur. $=\pi \times D \times (\lambda g + k\tau)$.		
	$\tau = 60$ cm.	$\tau = 40$ cm.	$\tau = 20$ cm.
1000 volts (or less)	0.8	0.9	1.1
2000 "	0.9	1.0	1.2
4000 "	1.0	1.1	1.4
6000 "	1.1	1.3	1.7
8000 "	1.2	1.5	2.0
10000 "	1.4	1.8	2.3
12000 "	1.7	2.0	2.8

For our design we have:

$$D = 62.0;$$
$$\lambda g = 43.0;$$
$$\tau = 32.5;$$
$$k = 0.9.$$

$\therefore$ " Equivalent radiating surface at air-gap "

$$= \pi \times 62.0 \times (43.0 + 29.3)$$
$$= \pi \times 62.0 \times 72.3$$
$$= 14\,000 \text{ sq.cm.}$$
$$= 140 \text{ sq.dm.}$$

The loss to be considered is, at full load:

11 630 watts.

Thus we have:

$$\text{Watts per sq.dm.} = \frac{11\,630}{140} = 83.4.$$

The data in Table 33, gives a rough notion of the temperature rise corresponding to various conditions.

TABLE 33.—DATA FOR ESTIMATING THE TEMPERATURE RISE IN INDUCTION MOTORS.

Peripheral Speed in Meters per Second.	Thermometrically determined ultimate temperature rise of open-protected types of induction motors, per watt of total loss per sq.dm. of equivalent radiating surface at the air-gap.		
	Badly Ventilated.	Fairly Ventilated.	Very Well Ventilated.
10	0.59	0.41	0.36
20	0.49	0.40	0.34
30	0.47	0.38	0.32
40	0.45	0.36	0.29
50	0.43	0.34	0.26
60	0.41	0.32	0.24

Our motor is decidedly well supplied with ventilating ducts, and with reasonable care in the arrangement of parts, it will come in the class of " very well ventilated " motors. The per-

ipheral speed has already (on page 113) been ascertained to be 16.2 meters per second.

From Table 33 we find that the value for the thermometrically determined temperature rise is:

$$0.35° \text{ per watt per sq.dm.}$$

This gives us a total (thermometrically determined) rise of

$$83.4 \times 0.35 = 29° \text{ Cent.}$$

Although in the predetermination of temperature rises, no close accuracy is practicable, still, the margin indicated by the above result is so great that the designer should rearrange the final design in such a manner as to save a little material at the cost of a slight increase in the losses.

Since a rise of fully 40° cent. is usually considered quite conservative (45° often being adopted), an **estimated** rise of 35° would leave a sufficient margin of safety.

THE WATTS PER TON

Another useful criterion to apply in judging whether a motor is rated at as high an output as could reasonably be expected, is the "watts per ton." The weight taken, is exclusive of slide rails and pulley. In the case of a design which has not been built, the method of procedure is as follows:

First estimate the weights of effective material.

Weight Stator Copper.

$$\text{mlt.} = 183.5 \text{ cm.}$$

Total number of turns	$= 3T = 3 \times 120 = 360;$
Cross-section stator cond	$= 0.302$ sq.cm.;
Volume of stator copper	$= 183.5 \times 360 \times 0.302$
	$= 19\,900$ cu.cm.;
Weight 1 cu.cm. of copper	$= 8.9$ grams;
Weight stator copper	$= \frac{19\,900 \times 8.9}{1000}$
	$= 177$ kg.

Weight Rotor Copper.

Total length rotor face conds (see page 175) $= 3290$ cm.;

Cross-section $= 5.4 \times 0.48 = 2.59$ sq.cm.;

Volume of rotor face conds $= 3290 \times 2.59$

$= 8550$ cu.cm.;

Weight rotor face conds $= \dfrac{8550 \times 8.9}{1000}$

$= 76$ kg.;

Mean circum. end ring $= 177$ cm.;

Cross-section $= 15.1$ sq.cm.;

Weight two end rings $= \dfrac{2 \times 177 \times 15.1 \times 8.9}{1000}$

$= 47.5$ kg.;

Total weight rotor copper $= 76 + 48 = 124$ kg.;

Total weight copper (stator plus rotor) $= 177 + 124$

$= 301$ kg.

Weight Stator Core.

This has already been estimated (on page 159) to be 603 kg.

Weight Rotor Core.

External diameter rotor core $= 61.7$ cm.;

Internal diameter rotor core $= 38.3$ cm.;

Gross area rotor core plate $= \dfrac{\pi}{4}(61.7^2 - 38.3^2)$

$= 1840$ sq.cm.;

Area 70 slots $= 70 \times 5.4 \times 0.48 = 180$ sq.cm.;

Net area rotor core plates $= 1660$ sq.cm.;

$\lambda_n = 27.9$ cm.;

Volume of sheet steel in rotor core $= 1660 \times 27.9$

$= 46\,300$ cu.cm.;

Weight of 1 cu.cm. of sheet steel $= 7.8$ grams;

Weight of rotor core $= \dfrac{46\,300 \times 7.8}{1000}$

$= 360$ kg.;

Total weight sheet steel (stator plus rotor) $= 603 + 360 = 963$ kg.

Thus we have:

Weight copper	= 301 kg.;
Weight sheet steel	= 963 kg.
Total net weight effec. material	= 1264 kg.

From experience in the design of motors of this type it has been determined that an adequate mechanical design is consistent with obtaining for the ratios of " Total Weight of Motor " to " Weight of Effective Material," the values given in Table 34.

TABLE 34.—DATA FOR ESTIMATING THE TOTAL WEIGHT OF AN INDUCTION MOTOR.

D — Diameter at Air-gap (in cm.).	Ratio of Total Weight (exclusive of slide rails and pulley) to Weight of Net Effective Material.	
	$D/\lambda g = 1.5$.	$D/\lambda g = 2.5$.
20	1.80	2.20
40	1.60	1.82
80	1.54	1.78

For our motor we have:

$$D = 62;$$

$$\lambda g = 43;$$

$$D/\lambda g = \frac{62}{43} = 1.44.$$

Total weight (exclusive of slide rails and pulley) $= 1.56 \times 1264$ $= 1970$ kg.

$= 1.97$ tons.

It will be remembered (p. 111) that our original rough estimate for the weight, was 2.0 tons. Even this last method, which depends upon obtaining a factor by which to multiply the calculated weight of effective material, can only be regarded as rough. We can, however, regard the value of 1.97 tons as fairly correct for the total weight of our motor.

Consequently we have:

$$\text{Watts per ton} = \frac{11\,630}{1.97} = 5900.$$

The designer must gain his own experience as to the attainable values for the " watts per ton." It may, however, here be stated that 8000 watts per ton and even considerably higher values are

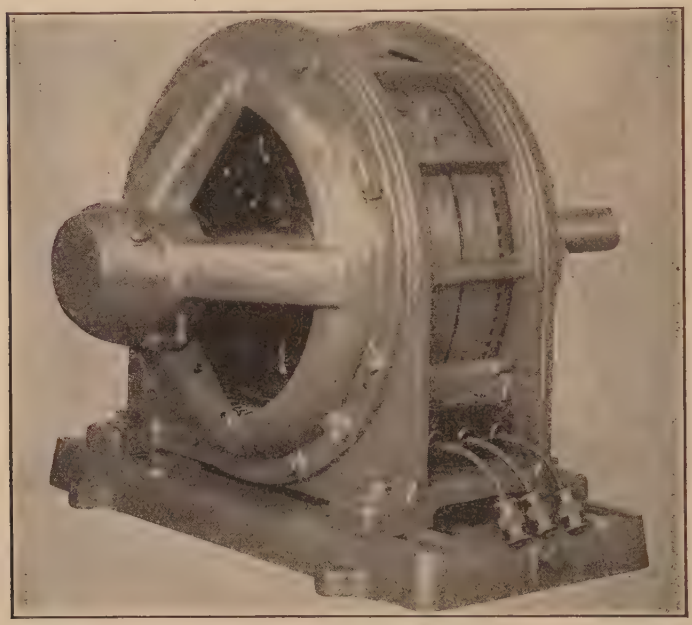


Fig 96.—An 8-pole, 50-H.P., 900-r.p.m., Squirrel-cage Induction Motor of the Open-protected Type. [Built by the General Electric Co. of America.]

quite consistent with moderate temperature rise for a motor of this size and speed, if constructed in the manner generally designated the " open-protected type." In Fig. 96 is given a photograph of an open-protected type of squirrel-cage induction motor.

The Breakdown Factor. It has been stated that so far as temperature-rise is concerned, the motor is rather liberally designed and that in a revised design it should be possible to save

a little in material. We must, however, take into account the limitation imposed by the heaviest load which the motor is capable of carrying before it pulls up and comes to rest. The ratio of this load to the rated load is termed the breakdown factor (**bdf.**) and the present custom is to require that a motor shall have a breakdown factor of about 2. In the case of our 200-h.p. motor, it ought to be capable of carrying an instantaneous load of $2\times200=400$ h.p. without pulling up and coming to rest.

It has previously been explained that the vertical projection of the vector representing the stator current in the circle diagram, is, for constant terminal pressure, proportional to the watts input to the motor. Thus the current corresponding to the maximum input is the current AM in the diagram in Fig. 97. The power input corresponding to this current may be represented by MN, the vertical radius of the circle. We have:

$$\begin{aligned} AM &= \sqrt{AN^2+MN^2} \\ &= \sqrt{\left(25.2+\frac{615}{2}\right)^2+\left(\frac{615}{2}\right)^2} \\ &= \sqrt{333^2+308^2} \\ &= \sqrt{206\,000} \\ &= 454 \text{ amperes.} \end{aligned}$$

Since the phase pressure is 577 volts, we have:

$$\begin{aligned} \text{Power input} &= 3\times577\times308 \\ &= 534\,000 \text{ watts.} \end{aligned}$$

Consequently when used as a measure of the power input, the vertical ordinates are to the scale of:

$$3\times577=1730 \text{ watts per ampere.}$$

But it is not the **input** which we wish, but the **output.** The I^2R losses at the full-load input of 102 amperes, amount to:

$$4540+3140=7680 \text{ watts.}$$

Consequently when the input is 454 amperes, the I^2R losses are:

$$\left(\frac{454}{102}\right)^2 \times 7680 = 152\,000 \text{ watts.}$$

The core losses and friction aggregate:

$$2410 + 240 + 1300 = 3950 \text{ watts.}$$

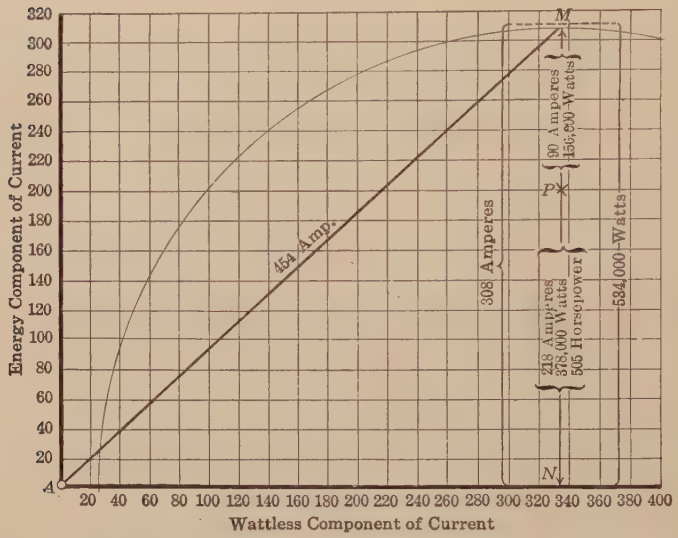


FIG. 97.—Diagram for Preliminary Consideration of the Breakdown Factor.

The total internal losses amount to:

$$152\,000 + 4000 = 156\,000 \text{ watts.}$$

Thus the **output** corresponds to that portion of the vertical radius MN, which is equal to:

$$534\,000 - 156\,000 = 378\,000 \text{ watts.}$$

This is the vertical height, PN, laid off equal to:

$$\frac{378\,000}{1730} = 218 \text{ amperes.}$$

This corresponds to an output of:

$$\frac{378\,000}{746} = 505 \text{ h.p.}$$

This vertical radius does not, however, correspond to the point of maximum output. Let us draw, as in Fig. 98, the vectors corresponding to:

430, 400 and 370 amperes.

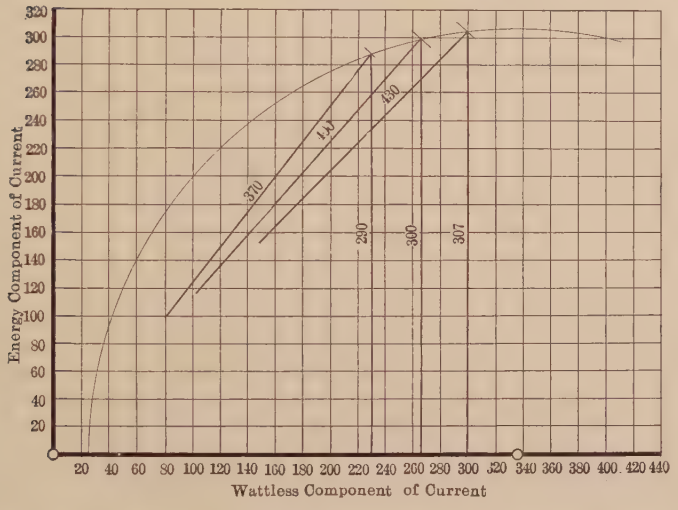


FIG. 98.—Revised Diagram for Determining the Breakdown Factor.

The corresponding vertical ordinates are:

307, 300 and 290 amperes.

The inputs are:

$$1730 \times 307 = 531\,000 \text{ watts},$$
$$1730 \times 300 = 520\,000 \text{ watts},$$
$$1730 \times 290 = 502\,000 \text{ watts}.$$

The I^2R losses are:

$$\left(\frac{430}{102}\right)^2 \times 7680 = 137\,000 \text{ watts},$$

$$\left(\frac{400}{102}\right)^2 \times 7680 = 118\,000 \text{ watts},$$

$$\left(\frac{370}{102}\right)^2 \times 7680 = 101\,000 \text{ watts}.$$

Adding the remaining losses of 4000 watts, we obtain for the total losses in the three cases:

141 000, 122 000, and 105 000 watts.

Deducting these losses from the respective inputs, we obtain as the three values for the output:

$$531\,000 - 141\,000 = 390\,000 \text{ watts},$$
$$520\,000 - 122\,000 = 398\,000 \text{ watts},$$
$$502\,000 - 105\,000 = 397\,000 \text{ watts}.$$

Obviously the output is a maximum at an input of 400 amperes and then amounts to:

$$\frac{398\,000}{746} = 534 \text{ h.p.}$$

Thus for our motor the bd.f. is:

$$\left(\frac{534}{200} = \right) 2.67.$$

A rough, empirical formula for obtaining the breakdown factor is:

$$bdf = \frac{0.4\gamma}{\sigma}.$$

For our motor we have:

$$\gamma = 0.248;$$

$$\sigma = 0.041.$$

$$\therefore \quad bdf = \frac{0.4 \times 0.248}{0.041}$$

$$= 2.42.$$

The use of this empirical formula is, in general, quite sufficient, as it is rarely of consequence to be able to estimate the bdf. at all closely.

THE POWER-FACTOR

It is a very simple matter to obtain a curve of power-factors for various values of the current input. In Fig. 99 are drawn

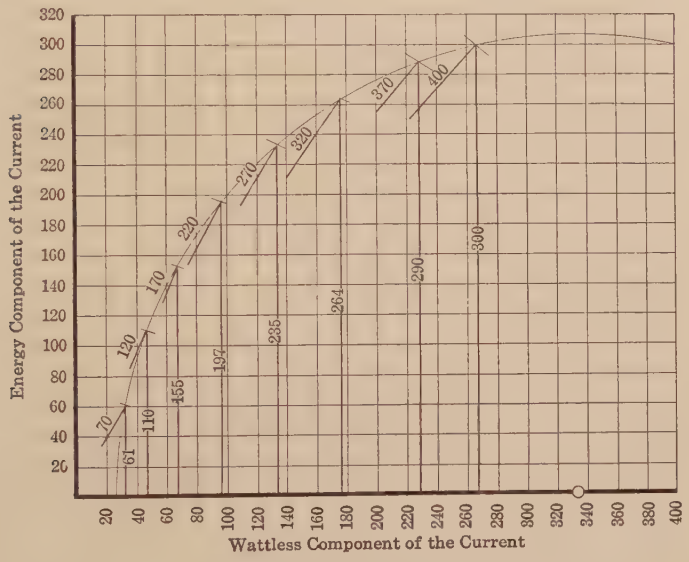


FIG. 99.—Diagram for Determining the Power-factors.

vectors representing stator currents ranging from the no-load current of 25.2 amperes, up to the break-down current of 400 amperes. In each case, the power-factor is the ratio of the vertical projection of the stator current, to the stator current. The values of the currents and of the vertical projections are recorded in Fig. 99, and also in the first two columns of the following table in which the estimation of the power-factor is carried out, and also the estimation of the efficiencies.

I. Stator Current per Phase.	II. Vertical Component.	III. Power Factor, (II ÷ I).	IV. Input in Watts (3 × 577 × II).	V. Losses.	VI. Output in Watts, (IV − V).	VII. Output in H.P.	VIII. Efficiency, $\left(\frac{VI \times 100}{IV}\right)$.
25.2	2.3	0.09	4 000	4 000	0	0	0
70	61	0.87	105 000	7 600	97 400	130	0.93
120	110	0.92	190 000	14 600	175 400	235	0.92
170	155	0.91	268 000	25 300	243 000	326	0.91
220	197	0.90	342 000	39 700	302 000	405	0.88
270	235	0.87	406 000	57 800	348 000	465	0.86
320	264	0.83	457 000	79 300	378 000	506	0.83
370	290	0.78	502 000	105 000	397 000	531	0.79
400	300	0.75	520 000	122 000	398 000	534	0.77

In Figs. 100 and 101, these values for the power-factor and efficiency are plotted as ordinates and with amperes per phase as abscissae. In Figs. 102 and 103 they are again plotted as ordinates but with the output in horse-power as abscissae.

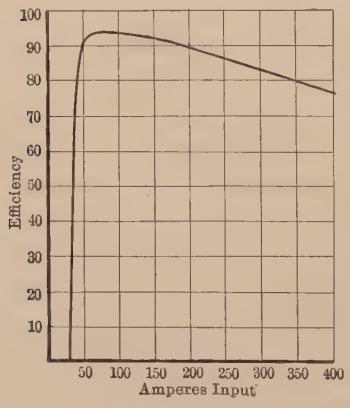


FIG. 100.—Efficiency Curve with Current Input as Abscissae.

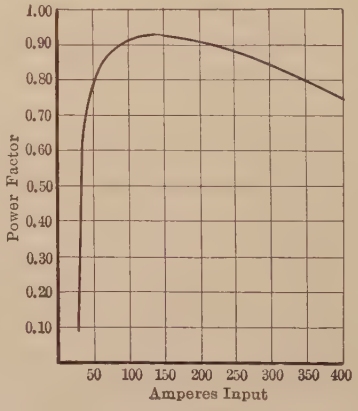


FIG. 101.—Power-factor Curve with Current Input as Abscissae.

THE SLIP

Were it not that the apparent resistance of the slot-embedded portions of the conductors of the squirrel cage, varies with the periodicity of the rotor currents, the determination of the slip at any load would be a very simple matter. The effect of this

variation is too small to be of consequence at ordinary loads and may for most purposes be neglected. For constant apparent resistance, percentage slip is the percentage which the rotor I^2R loss constitutes of the input to the rotor.

The Determination of the Rotor I^2R Loss at Various Loads. By constructions similar to that illustrated in Fig. 94 on p. 179, we may obtain the values of the rotor current for various values of the stator current. A number of such values is tabulated below:

Stator Current.	Rotor Current for a 1 : 1 Ratio.	Actual Rotor Current (Ratio of Trans. is 10.3 : 1).
25.2 amp.	0 amp.	0 amp.
70 "	62 "	640 "
120 "	113 "	1160 "
170 "	162 "	1670 "
220 "	210 "	2160 "
270 "	256 "	2640 "
320 "	305 "	3140 "
370 "	354 "	3640 "
400 "	384 "	3960 "

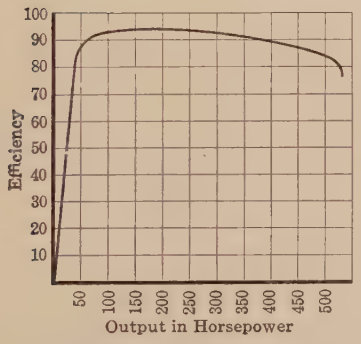


FIG. 102.—Efficiency Curve with Horse-power Output as Abscissae.

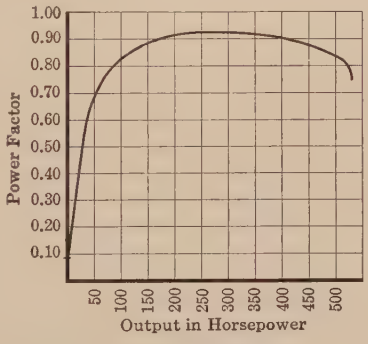


FIG. 103.—Power-factor Curve with Horse-power Output as Abscissae.

With the full-load stator current of 102 amperes we have seen (on p 166.) that the squirrel-cage loss is 3140 watts. From the above table we see that for a stator current of 102 amperes, the rotor current is $\left(\frac{102}{120}\times 1160 = \right)$ 985 amperes. Thus, referred

to the current in the face conductors, the squirrel cage may be considered as having an " equivalent " resistance of:

$$\frac{3140}{985^2} = 0.00324 \text{ ohm.}$$

In the following table, the rotor I^2R losses, the input to the rotor, the slip and the speed have been worked out:

Stator Current. [A]	Stator I^2R Loss, $(3 \times 0.146 \times A)$. [B]	Stator Core Loss. [C]	Input to Motor in Watts (from p. 192). [D]	Input to Rotor in Watts (D–B–C). [E]	Current in Rotor Face Conds. (from p. 193). [F]	Rotor I^2R Loss, $(0.00324 \times F^2)$. [G]	Rotor Slip in Per Cent. $\left[\frac{G}{E} \times 100\right]$	Output from Motor in h.p.	Speed of Motor.
25.2	280	2410	4 000	1 310	0	0	0	0	500
70	2 140	2410	105 000	100 450	640	1 330	1.32	130	493
120	6 300	2410	190 000	181 300	1160	4 350	2.40	235	488
170	12 600	2410	268 000	253 000	1670	9 000	3.55	326	482
220	21 100	2410	342 000	318 500	2160	15 100	4.75	405	476
270	32 000	2410	406 000	371 600	2640	22 500	6.05	465	469
320	44 800	2410	457 000	409 800	3140	32 000	7.81	506	461
370	60 000	2410	502 000	439 600	3640	43 000	9.80	531	451
400	70 000	2410	520 000	447 600	3960	51 000	11.4	534	443

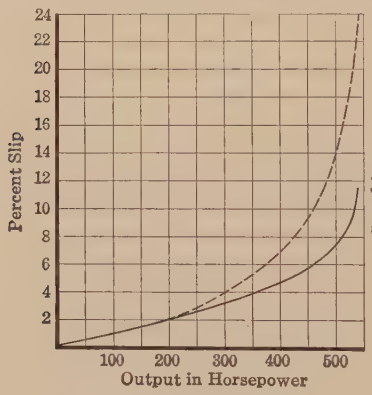


Fig. 104.—Curves of Slip.

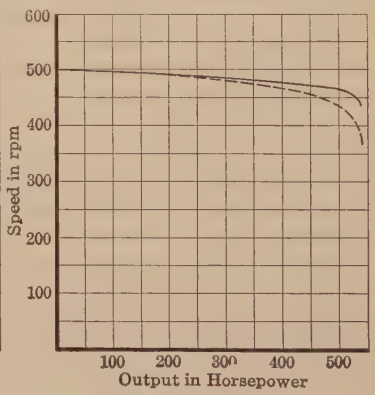


Fig. 105.—Curves of Speed.

These values for " slip " and speed are plotted in the full-line curves in Figs. 104 and 105; the dotted line curves being added to indicate **qualitatively** the general way in which the results would be affected by the influence of the frequency of the rotor currents in increasing the apparent resistance of the slot-embedded portions of the squirrel-cage.

CHAPTER V

SLIP-RING INDUCTION MOTORS

Probably considerably over 90 per cent of the total number of induction motors manufactured per annum are nowadays of the squirrel-cage type. The strong preference for this type is on account of the exceeding simplicity of its construction and the absence of moving contacts. Nevertheless occasions arise when it is necessary to supply definite windings on the rotor and to connect these up to slip rings. Sometimes this is done in order to control the starting torque and to obtain a higher starting torque for a given stator current than could be obtained with a squirrel-cage motor and sometimes the use of slip rings is for the purpose of providing speed control by regulating external resistances connected in series with the rotor windings. This is a very inefficient method of providing speed control, but cases occasionally arise when it is the economically correct method to employ.

Since the controlling resistance is located external to the motor, there is no longer occasion to study to provide a deep rotor conductor in order to obtain a desired amount of starting torque. With freedom from this restriction, the density at the root of the rotor tooth would (for a design of the rating we have discussed in Chapter IV) be taken higher than the 15 500 lines per sq. cm. adopted for the squirrel-cage design. We shall do well to employ a somewhat shallower and wider slot, for it must be observed that it now becomes necessary to provide space for insulating the rotor conductors.

Let us employ 70 slots, as before, but let the slots have a depth of only 30 mm. and a (punched) width of 8 mm. The diameter at the bottom of the slots will now be:

$$617.4 - 2 \times 30 = 557.4 \text{ mm.}$$

The rotor slot pitch is now:

$$\frac{557.4\times\pi}{70}=25.0 \text{ mm.}$$

$$\text{Width tooth at root}=25.0-8.0=17.0 \text{ mm.}$$

For the squirrel-cage design, the slot pitch (see p. 888) was 22.8 mm. and the width of tooth at the root, was:

$$22.8-4.8=18.0 \text{ mm.}$$

Thus the tooth density will be increased in the ratio of 17 to 18, but since the length of the tooth has been decreased in the ratio of 54 to 30, the mmf. required for the rotor teeth will be no greater than for the squirrel-cage design.

The thickness of the slot insulation will be 0.8 mm.

Allowing 0.3 mm. tolerance in assembling the punchings, the assembled width of the slot will be:

$$8.0-0.3=7.7 \text{ mm.}$$

After deducting the portions of the width occupied by the slot insulation on each side, we arrive at the value of:

$$7.7-2\times0.8=6.1 \text{ mm.}$$

for the width of the conductors, of which there will be 2 per slot, arranged one above the other. Of the depth of 30 mm., the insulation and the wedge at the top of the slot will require a total allowance of 6 mm. Consequently for the depth of each conductor we have:

$$\left(\frac{30-6}{2}=\right) 12 \text{ mm.}$$

Thus each conductor has a height of 12 mm., a width of 6.1 mm. and a cross-section of:

$$12\times6.1=73 \text{ sq.mm.}$$

A drawing of the slot is given in Fig. 106.

A two-layer, full-pitch, lap winding will be employed. There are 70 slots and (2×70=) 140 conductors. The winding will be of the 2-circuit type. It is impracticable in this treatise to deal with the extensive subject of armature windings. The reader will find a discussion of the laws of 2-circuit windings on page 3 of the Author's "Elementary Principles of Continuous-Current Dynamo Design" (Whittaker & Co., London and New York), and also in Chapter VIII of Hobart and Ellis' "Armature Construction" (Whittaker & Co.). It must suffice here to state that such a winding is usually arranged to comply with the formula:

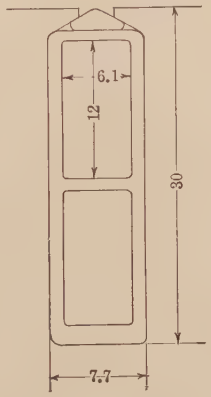


FIG. 106.—Slot of "Wound" Rotor.

$$F = Py \pm 2.$$

In this formula, the total number of conductors is denoted by F, the number of poles by P, and the "winding pitch" by y. The conductors are numbered from 1 to 140 as indicated in the diagram in Fig. 107. The winding pitch, y, is that quantity which is added to the number of any conductor in order to ascertain the number of the conductor to which the first conductor is connected.

For our case we have:

$$140 = 6y \pm 2;$$

$$y = \frac{138}{6} = 23.$$

Consequently conductor number 1 is connected (as indicated in Fig. 107), to conductor number (1+23=) 24; conductor number 24 to conductor number (24+23=) 47, etc., until the entire 140 conductors are interconnected to constitute a singly-reentrant circuit. After the diagram has been carried out in this way in pencil, the next step consists in dividing up the $\left(\frac{140}{2} = \right)$ 70 turns into six separate circuits. The six circuits

cannot contain equal numbers of turns. The nearest to this consists in subdividing the winding up into two circuits each comprising 22 conductors and four circuits each comprising 24 conductors. We have

$$22+22+24+24+24+24=140.$$

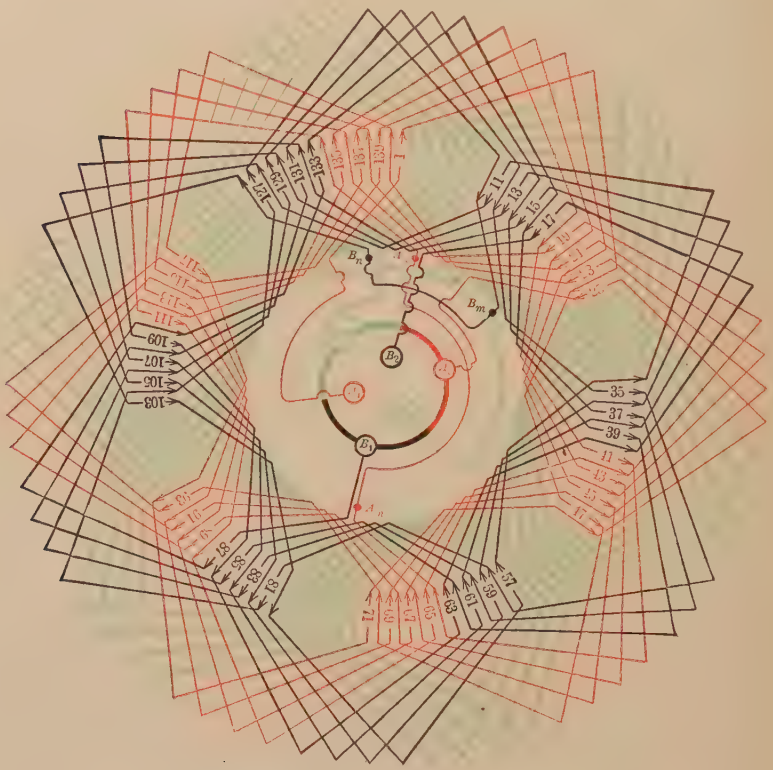


FIG. 107.—Winding Diagram for the Rotor of a Slip-ring Induction Motor.

Let us start in with conductor number 1 and designate this beginning by A_1, as shown in Fig. 107. After following through the 22 conductors:

(1, 24, 47, 70, 93, 116, 139, 22, 45, 68, 91, 114, 137,
20, 43, 66, 89, 112, 135, 18, 41, 64),

we interrupt the winding and bring out a lead at the point Am. We then start in again, indicating the point as B_1 and proceed next through conductor number (64+23=) 87. After passing through 22 more conductors we come out again at a point which we designate as Bm. The next circuit starts at C_1 and ends at Cm. The remaining three circuits are A_2A_n, B_2B_n and C_2C_n.

We now have six circuits. An inspection will show that these can be grouped in three pairs which have, in Fig. 107 been drawn in red, black and green. The red phase comprises A_1A_m connected in series with A_nA_2 forming A_1A_2. In the same way, the black phase B_1B_2 is obtained, and the green phase C_1C_2. By considering the instant when the current in Phase A is of the value 1.00 and is flowing from the line toward the common connection, while the currents in Phases B and C are of the value 0.50 and are flowing from the common connection to the lines, it is ascertained that the ends A_2, B_1 and C_2 must be brought together to form the common connection, the ends A_1, B_2 and C_1 constituting the terminals of the motor.

For the mean length of turn of this rotor winding we have the formula:

$$\begin{aligned} \text{mlt.} &= 2\lambda g + 2.5\tau; \\ \lambda g &= 43; \\ 2\lambda g &= 86; \\ \tau &= 32.5; \\ 2.5\tau &= 82; \\ \text{mlt.} &= 86+82 = 168 \text{ cm.} \end{aligned}$$

The entire winding comprises (70×2=)140 conductors and 70 turns. Consequently the average length per phase is:

$$\frac{70\times 168}{3} = 3920 \text{ cm.}$$

$$\text{Resis. per phase (at 60° Cent.)} = \frac{3920\times 0.0000020}{0.73} = 0.0107 \text{ ohm.}$$

Since there are 720 stator conductors, the ratio of transformation is now:

$$\left(\frac{720}{140} = \right) 5.15 \text{ to } 1.00.$$

Referring to p. 165, we see that for a 1:1 ratio, the rotor current corresponding to the full-load stator current of 102 amperes, is 96 amperes. Consequently the full-load rotor current for our wound rotor, is:

$$5.15 \times 96 = 495 \text{ amperes.}$$

The full load I^2R loss in the rotor is:

$$3 \times 495^2 \times 0.0107 = 7800 \text{ watts.}$$

This is $\left(\frac{7800}{3140} = \right)2.5$ times the loss in our squirrel-cage design. The increase is due partly to the waste of space for slot insulation and partly to the long end connections inevitably associated with a " wound " rotor.

These long end connections also involve additional magnetic leakage at load, and the circle ratio of a motor with a "wound" rotor may usually be estimated on the basis of a 25 per cent increase in the values in Table 27, on pp. 150 and 151. Consequently for the present design we have:

$$\sigma = 1.25 \times 0.041 = 0.051.$$

We still have (as on p. 140), $\gamma = 0.242$.

Hence for the circle diameter we now have:

$$\text{Dia. of circle} = \frac{0.242 \times 102}{0.051} = 485 \text{ amperes}$$

The " ideal " short-circuit current is now:

$$0.242 \times 102 + 485 = 510 \text{ amperes.}$$

as against the value of 645 amperes for the squirrel-cage design.

We now have:

$$bdf = \frac{0.4\gamma}{\sigma} = \frac{0.4 \times 0.242}{0.051} = 1.9.$$

From an examination of these various ways in which the properties of the motor have suffered by the substitution of a wound rotor, it will be understood that the squirrel-cage type is much to be preferred, and that every effort should be made to employ it when the conditions permit. In addition to the defects noted, it must not be overlooked that the efficiency and power-factor of the slip-ring motor are lower and the heating greater. Furthermore the squirrel-cage motor is more compact and is lighter and cheaper. It will also be subject to decidedly less depreciation, and can be operated under conditions of exposure which would be too severe for a slip-ring motor.

CHAPTER VI

SYNCHRONOUS MOTORS VERSUS INDUCTION MOTORS

The circumstance that a type of apparatus, possesses particularly attractive features, is liable to lead occasionally to disappointment, owing to its use under conditions for which it is rendered inappropriate by its possession of other less-well-known features, which, under the conditions in question, are undesirable.

While the squirrel-cage induction motor is of almost ideal simplicity, there are many instances in which its employment would involve paying dearly for this attribute. It is, for instance, a very poor and expensive motor for low speeds. When the rated speed is low, and particularly when, at the same time, the periodicity is relatively high, the squirrel-cage motor will inevitably have a low power-factor. In Figs. 108 and 109 are drawn curves which give a rough indication of the way in which the power-factor of the polyphase induction motor varies with the speed for which the motor is designed. While *qualitatively* identical conclusions will be reached by an examination of the data of the product of any large manufacturer, the *quantitative* values may be materially different, since each manufacturer's product is characterized by variations in the degree to which good properties in various respects are sacrificed in the effort to arrive at the best all-around result. It is for this reason that instead of employing data of the designs of any particular manufacturer, the curves in Figs. 108 and 109 have been deduced from the results of an investigation published by Dr. Ing. Rudolf Goldschmidt, of the Darmstadt Technische Hochschule, in an excellent little volume which he has published under the title: "Die normalen Eigenschaften elektrischer Maschinen" (Julius Springer, Berlin). Whereas from the 50-cycle curves of Fig. 109, it will be seen that, at low speeds, motors of from 50 to 500 h.p.

rated output, have power-factors of less than 0.80, the curves in Fig. 108 show that the power-factors of equivalent 25-cycle motors are distinctly higher.

It is desirable again to emphasize that the precise values employed in the curves in Figs. 108 and 109, have, considered individually, no binding significance. Thus, by sacrificing desirable features in other directions and by increased outlay in the construction of the motor, slightly better power-factors may sometimes be obtained. On the other hand, the power-factors on which the curves are based, are, in most instances, already representative of fairly extreme proportions in this respect; and few manufacturers find it commercially practicable to list low-speed motors with such high power-factors as are indicated by these curves. The purchaser would rarely be willing to pay a price which would leave any margin of profit were these power-factors provided; and consequently it is only relatively to one another that these curves are of interest. They teach the lesson that for periodicities of 50 or 60 cycles, it must be carefully kept in mind that, if low speed induction motors are used, either the price paid must be disproportionately high, or else the purchaser must be content with motors of exceedingly low power-factors. The power-factor, furthermore, decreases rapidly for a given low speed, with decreasing rated output.

On the other hand, for a 25-cycle supply, these considerations are of decidedly less importance. With a clear recognition of this state of affairs, a power user desiring low-speed motors for a 60-cycle circuit will do well to take into careful consideration the alternative of employing *synchronous* motors. If he resorts to this alternative he can maintain his power-factor at unity, irrespective of the rated speed of his motors; but he must put up with the slight additional complication of providing for a rotor which, in addition to the squirrel-cage winding, is also equipped with field windings excited through brushes and slip-rings from a source of continuous electricity. The precise circumstances of any particular case will require to be considered, in order to decide whether or not this alternative is preferable.

While it has long been recognized that the inherent simplicity and robustness of the squirrel-cage induction motor constitute features of very great importance and justify the wide use of such motors, nevertheless we must not overlook the fact that

induction motors are less satisfactory the lower the rated speed. The chief disadvantage of a low-speed induction motor is its low power-factor as shown above. Let us take the case of a 100-h.p., 60-cycle motor. The design for a rated speed of 1800 r.p.m. will (see Fig. 109) have a power-factor of 93 per cent, whereas the design for a 100-h.p., 60-cycle motor for one-tenth of this speed, i.e., for a speed of only 180 r.p.m., will have a power-factor of only some 76 per cent. The power-factors given above correspond to rated load. For light loads, the inferiority of the low-speed motor as regards low power-factor is much greater, and this circumstance further accentuates the objection to the use of such a motor.

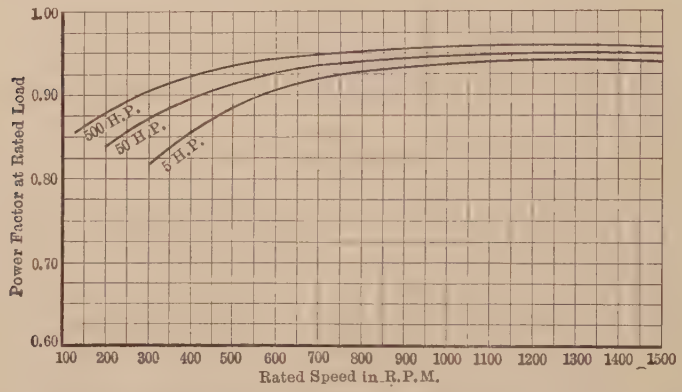


Fig. 108.—Curves of Power-factors of 25-cycle Induction Motors of 5 H.P., 50 H.P., and 500 H.P. Rated Output, Plotted with Rated Speeds as Abscissae.

For a long time there has existed a general impression that a synchronous motor could not be so constructed as to provide much starting torque. Otherwise it would probably have been realized that, for low speeds, it would often be desirable to give the preference to the synchronous type. For with the synchronous type, the field excitation can be so adjusted that the power-factor shall be unity; indeed there is no objection to running with *over*-excitation and reducing the power-factor again below unity, thus occasioning a consumption of *leading* current by the synchronous motor.

By the judicious admixture (on a single supply system) of

high-speed induction motors consuming a slightly-lagging current and low-speed synchronous motors consuming a slightly leading current, it is readily feasible to operate the system at practically unity power-factor; and thereby to obtain, in the generating station and on the transmission line, the advantages usually accruing to operation under this condition.

Now the point which is beginning to be realized, and which it is important at this juncture to emphasize, is that the synchronous motor, instead of being of inferior capacity as regards the provision of good starting torque, has, on the contrary, certain inherent attributes rendering it entirely feasible to equip it for much more liberal starting torque than can be provided by

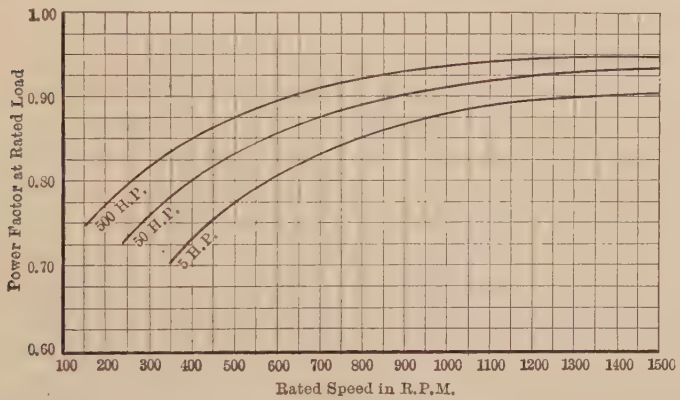


FIG. 109.—Curves of Power-factors of 50-cycle Induction Motors of 5 H.P., 50 H.P., and 500 H.P. Rated Output, Plotted with Rated Speeds as Abscissae.

efficient squirrel-cage induction motors. The word "efficient" has been emphasized in the preceding statement and for the following reason: by supplying an induction motor with a squirrel-cage system composed of conductors of sufficiently small cross-section, and consequently sufficiently-high resistance, any amount of starting torque which is likely to be required, can be provided. Unfortunately, however, the high-resistance squirrel cage is inherently associated with a large I^2R loss in the rotor when the motor is carrying its load. This large I^2R loss not only occasions great "slip" but also occasions very low efficiency. Furthermore, owing to this very low efficiency,

the heating of the motor would be very great were it not that such a motor is rated down to a capacity far below the capacity at which it could be rated were it supplied with a low resistance (i.e., low starting torque) squirrel-cage system.

But when we turn to the consideration of the *synchronous* motor, we note the fundamental difference, that the squirrel-cage winding with which we provide the rotor, is only active during starting, and during running up toward synchronous speed. When the motor has run up as far toward synchronous speed as can be brought about by the torque supplied by its squirrel cage, excitation is applied to the field windings and the rotor pulls in to synchronous speed. So soon as synchronism has been brought about, the squirrel-cage system is relieved of all further duty; and it is consequently immaterial whether it is designed for high or for low resistance.

Consequently with the *synchronous* motor we are completely free from the limitations which embarrass us in designing high-torque *induction* motors. In the case of the synchronous motor, we can provide any reasonable amount of torque by making the squirrel-cage system of sufficiently high resistance. One difficulty, however, presents itself: while, at the instant of starting, we may desire very high torque and may provide it by a high-resistance squirrel cage, the higher the resistance the more will the speed, up to which the rotor will be brought by the torque of the squirrel cage, fall short of synchronous speed. It would in fact, be desirable that as the rotor acquires speed the squirrel cage should gradually be transformed from one of high resistance to one of low resistance. If we could accomplish the result that the squirrel cage should, at the moment the motor starts from rest, have a **high** resistance, and if it could be arranged that this resistance should gradually die away to an exceedingly **low** resistance as the motor speeds up, then the motor would gradually run up to practically synchronous speed and would furnish ample torque throughout the range from zero to synchronous speed.

We can provide precisely this arrangement if, instead of making the end rings of the squirrel cage, of copper or brass or other non-magnetic material, we employ instead, end rings of *magnetic* material, such as wrought iron, mild steel, cast iron or some magnetic alloy. Let us consider the reason why this arrangement should produce the result indicated. Just before the motor starts, the

currents induced in the squirrel-cage system are of the full periodicity of the supply. In the end rings, these currents will, with usual proportions, be very large in amount; and since the currents are alternating and since the material of the end rings is magnetic, there will be a very strong tendency, in virtue of the well-known phenomenon generally described as "skin effect," to confine the current to the immediate neighborhood of the surface of the end rings. The current will be unable to make use of the full cross-section of the end rings; and consequently, even though the end rings may be proportioned with very liberal cross-section, the net result will, at starting, be the same as if the end rings were of high resistance. But as the motor speeds up, the periodicity of the currents in the squirrel cage decreases, until, at synchronism, the periodicity would be zero and there would be no "skin effect." In view of these explanations, it is obvious that the impedance of the end rings will gradually decrease from a high value at starting to a low value at synchronism.

Now we are more free to make use of this phenomenon in the case of synchronous motors than in the case of induction motors, for as previously stated, when the synchronous motor is run at full speed, the squirrel cage is utterly inactive (except in serving to minimize "surging" and to decrease "ripple" losses); whereas the induction motor's squirrel cage is always carrying alternating current (even though of low periodicity); and this alternating current flowing through end rings of magnetic material, occasions a lower power-factor than would be the case with the equivalent squirrel-cage motor with end rings of non-magnetic material. Even in the case of induction motors, excellent use can be made of constructions with end rings of magnetic material, in improving the starting torque, but there is unavoidably at least a little sacrifice in power-factor during normal running.

It should now be clear that there is a legitimate and wide field for low-speed synchronous motors and that these motors will be superior to low-speed induction motors, in that, while the former can be operated at unity power-factor, or even with leading current if desired, the latter will unavoidably have very low power-factors. Furthermore these low-speed synchronous motors, instead of being in any way inferior to induction motors with respect to starting torque, have attributes permitting of providing them with higher starting torque than can be provided with

induction motors, without impairing other desirable characteristics such as low heating and high efficiency.

Of course there always remains the disadvantage of requiring a supply of continuous electricity for the excitation of the field magnets. Cases will arise where this disadvantage is sufficient to render it preferable, even for low-speed work, to employ

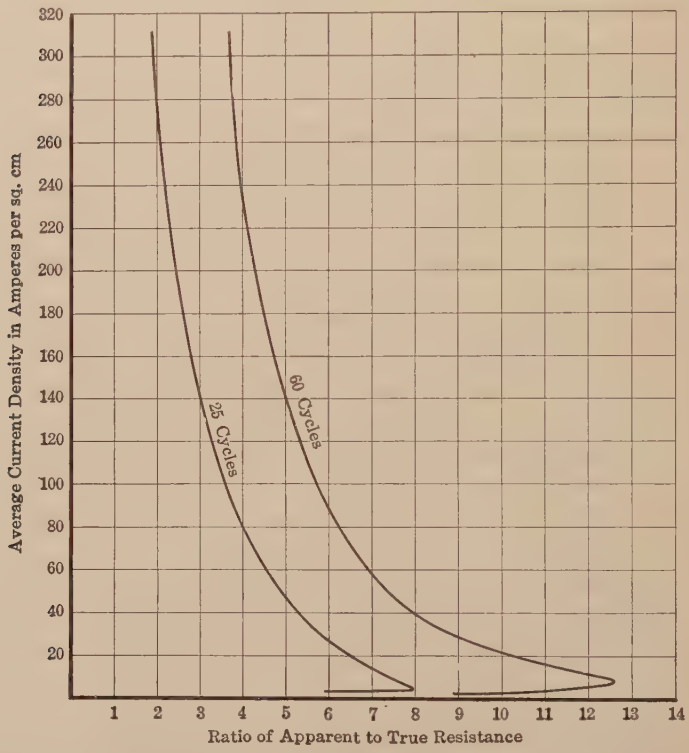


FIG. 110.—Robinson's Curves for Skin Effect in Machine-steel Bar with a Cross-section of 25×25 mm.

induction motors, but in the majority of cases where polyphase motors must operate at very low speeds, it would appear that synchronous motors are preferable.

In the curves in Figs. 110 and 111 are plotted the results of some interesting tests which have recently been made by Mr. L. T. Robinson, on "skin effect" in machine steel bars. By

means of these and similar data, and by applying to the design of the synchronous motor the ample experience which has been acquired in the design of induction motors, the preparation of a design for a synchronous motor for stipulated characteristics as regards torque, presents no difficulties. The smooth-core type of field with distributed excitation is to be preferred to the salient-pole type.

We have considered the relative inappropriateness of the induction motor for low-speed applications. Conversely it is a particularly excellent machine for high speeds. Its power-

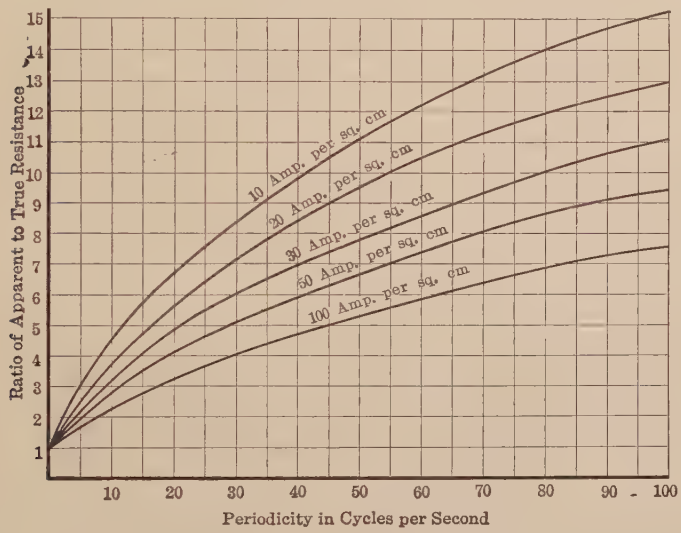


FIG. 111.—Robinson's Curves for Skin Effect in Machine-steel Bar with a Cross-section of 25×25 mm.

factor is higher the higher the rated speed; and when we come to very high speeds we obtain, in motors of large capacity, full-load power-factors in excess of 95 per cent. In such instances the simplicity of squirrel-cage induction motors should frequently lead to their use in preference to synchronous motors.

Reasoning along similar lines, in the case of generators, the induction type offers advantages over the synchronous type in many instances. It cannot, however, *replace* the synchronous generator even at high speeds, for as explained in Chapter VII, it requires to be run in parallel with synchronous generators, the

latter supplying the magnetization for the induction generators and also supplying the lagging component of the external load when the latter's power-factor is less than unity. Notwithstanding these limitations, there is a wide field for the induction generator; and the above indications may be useful for guidance in showing its appropriateness in any specific instance.

It will now be agreed that the properties of synchronous motors lend themselves admirably to the provision of high starting torque. In addition to the means previously described whereby a synchronous motor may have not only high starting torque but may also automatically run close up to synchronous speed, so as to fall quietly into synchronism immediately upon the application of the field excitation from the continuous-electricity source, there is also available a phenomenon described by Mr. A. B. Field, in a paper entitled "Eddy Currents in Large Slot-Wound Conductors," presented in June, 1905, before the American Institute of Electrical Engineers (Vol. 24, p. 761).

Mr. A. B. Field analyzed the manner in which the apparent resistance of slot-embedded conductors varies with the periodicity. Subsequent investigations show that the Field phenomenon, while harmful in stator windings exposed constantly to the full periodicity of the supply system, may be employed to considerable advantage in the proportioning of the conductors of the slot portion of a squirrel-cage system. Both in synchronous and in induction motors, this is an important step. Take, for instance, the case of a 60-cycle induction motor. At starting, the periodicity of the currents in the squirrel-cage system is 60 cycles, and the Field effect may, with properly-proportioned conductors, be sufficient to occasion an apparent resistance very much greater than the true resistance. The motor thus starts with very much more torque per ampere, than were the Field phenomenon absent. As the motor speeds up, the periodicity decreases and the Field effect dies out, the apparent resistance of the squirrel-cage gradually dying down to the true resistance. Thus, whereas the full-load running slip of an ordinary squirrel-cage motor must be high, its heating high and its efficiency low, if it is to develop high starting torque, we may, by using the Field effect, construct high-starting torque motors with low slip and high efficiency.

In Fig. 112 are shown respectively the rough characteristic shape of a curve in which torque is plotted as a function of

the speed for : *A*, a permanently low-resistance squirrel-cage motor; *B*, a permanently high-resistance squirrel-cage motor; and, *C*, for a squirrel-cage motor in which the apparent resistance gradually changes from a high to a low value as the motor runs up from rest to synchronism. As regards the application of these curves to a synchronous motor, it will be seen that, in the first case, (Curve *A*), the starting torque is rather low; but that the torque increases, passes through a maximum, and falls slowly, remaining quite high until the speed is close to synchronism. This motor, while unsatisfactory at starting, has the property

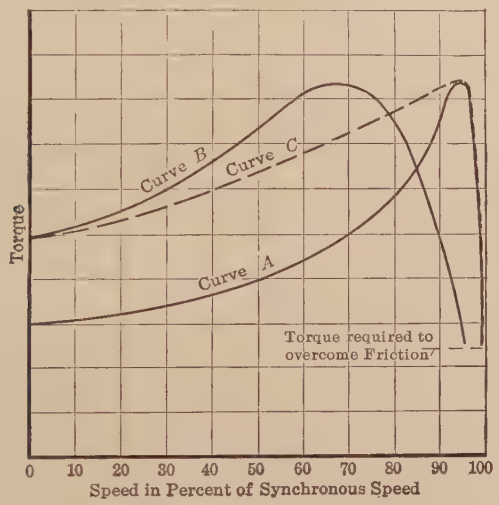


FIG. 112.—Curves Contrasting Three Alternative Squirrel-cage Constructions for a Synchronous Motor.

of pulling easily into synchronism on the application of the excitation from the continuous-electricity source. In the second case, (Curve *B*), while the synchronous motor *starts* with high torque, the torque falls away much more rapidly; and when the torque has fallen to the value necessary to overcome the friction of the motor, the speed is several per cent below synchronous speed, and the application of the continuous excitation, if it suffices to pull the rotor into synchronism, will do so only at the cost of an abrupt and considerable instantaneous drain of power from the line. In the third case, (Curve *C*), there are present

the good attributes of the first two cases, the bad attributes being completely eliminated.

It is thought that with the explanations furnished in this Chapter, the user will be assisted in determining, in any particular case, whether it is more desirable to take advantage of the extreme simplicity and toughness of the low-speed squirrel-cage induction motor, notwithstanding its poor power-factor, or whether he should employ the slightly more complicated synchronous motor in order to have the advantage of high power-factor.

Another way of dealing with the situation, which in certain cases of a low-speed drive is preferable, is to employ a high-speed induction motor (which will consequently have a high power-factor), and to gear it down to the low-speed load. Thus, from the curves in Fig. 109, we see that, if we require to drive a load at 200 r.p.m., a 50-h.p. motor will have a power-factor of only about 0.70 ; whereas, if the motor were to drive the load through 5 to 1 gearing, the motor's own speed would be 1000 r.p.m., and its power-factor would be over 0.90. Not only would the induction motor be characterized by a 20 per cent higher power-factor at full load, but its efficiency would be higher and it would be much smaller, lighter and cheaper. At half load the advantage of the high-speed motor in respect to power-factor is still more striking, the two values being of the following order:

Rated Speed.	Power-factor at Half Load.
200 r.p.m.	0.50
1000 r.p.m.	0.78

CHAPTER VII

THE INDUCTION GENERATOR

For the very high speeds necessary in order to obtain the best economy from steam turbines, the design of synchronous generators is attended with grave difficulties. The two leading difficulties relate, first to obtaining a sound mechanical construction at these high speeds, and, secondly, to the provision of a field winding which shall operate at a permissibly low temperature.

It is very difficult to provide adequate ventilation for a rotating field magnet of small diameter and great length. The result has been that for so high a speed as 3600 r.p.m., the largest synchronous generators which have yet given thoroughly satisfactory results are only capable of a sustained output of some 5000 kva. Even at this output the design has very undesirable proportions and the temperature attained by the field winding is undesirably high.

But an **Induction** generator is exempt from the worst of these difficulties. The conducting system carried by its rotor consists in a simple squirrel cage, the most rugged construction conceivable. Thus from the mechanical standpoint the induction generator is an excellent machine. Furthermore, the squirrel cage can be so proportioned that the full-load I^2R loss is exceedingly low. Hence the rotor will run cool, and is, in this respect, in striking contrast with the rotor of a high-speed **synchronous** generator.

Notwithstanding these satisfactory attributes, induction generators are rarely employed. The chief obstacle to their extensive use relates to the limitation that they must be operated in parallel with **synchronous** apparatus. **Induction** generators are dependent for their excitation upon lagging current drawn from **synchronous** generators, or leading current delivered to **synchronous** motors connected to the network into which the induction generators deliver their electricity.

The practical aspects of the theory of the induction generator have been considered in a paper (entitled "The Squirrel-cage Induction Generator") written in collaboration by Mr. Edgar Knowlton and the present author. This paper was presented on June 28, 1912, at the 29th Annual Convention of the American Institute of Electrical Engineers at Boston. It would not be appropriate to reproduce the descriptions and explanations in that paper, since the purpose of the present treatise is to set forth the fundamental methods of procedure employed in designing machinery. Some brief extracts from the paper in question are given in a later part of this chapter, and the reader will find it profitable to consult the original paper in amplifying his knowledge of the practical aspects of the theory of the induction generator.

Although the induction generator is chiefly suitable for large outputs, we can nevertheless, in explaining designing methods, employ as an illustrative example the case of adapting to the purposes of an induction generator, the 200-h.p. squirrel-cage induction motor which we worked out in Chapter IV.

The full-load efficiency has (see p. 178) been ascertained to be 93.0 per cent. Consequently at full load the input is :

$$\frac{200 \times 746}{0.930} = 160 \text{ kw.}$$

Let us, in the first instance, assume that the machine is suitable for a rated output of 160 kw. when operated as an induction generator. Since, when employed for this purpose, there is no longer any necessity for taking into consideration any questions relating to starting torque, let us reduce the rotor I^2R loss by widening the rotor face conductors. In the original design of the induction motor the slip corresponding to 200 h.p. was 2.0 per cent. This slip corresponds to a rotor I^2R loss of 3140 watts. This loss is made up of two components, which are 2512 watts in the face conductors and 628 watts in the end rings.

The tooth density in the rotor is needlessly low, other considerations, not entering into induction **generator** design having determined the width of the slot. There is now nothing to prevent doubling the width of the rotor conductors. This gives

for the dimensions of these conductors a depth of 54 mm. and a width of 9.6 mm.

This alteration will reduce the rotor loss at 160 kw. output to

$$\frac{2512}{2}+628=1884 \text{ watts.}$$

The slip will now be:

$$\frac{1884}{3140}\times 2.0=1.2 \text{ per cent.}$$

In the design of an induction generator, the slip should be made as small as practicable, in order to have a minimum rotor loss and consequently the highest practicable efficiency and low heating. In large induction generators there is rarely any difficulty in bringing the slip down to a small fraction of 1 per cent. The slip of the 7500-kw. 750-r.p.m. induction generators in the Interborough Rapid Transit Co.'s 59th Street Electricity Supply Station in New York, is only about three-tenths of 1 per cent at rated load.

Since we have reduced the squirrel-cage loss by:

$$(2.0-1.2=)0.8$$

per cent, the efficiency of our 160-kw. induction generator will be

$$(93.0+0.8=)93.8 \text{ per cent.}$$

It is not this small increase in efficiency which is particularly to be desired, but the decreased heating. The total losses are decreased from:

$$(100.0-93.0=)7.0 \text{ per cent of the input}$$

to

$$(100.0-93.8=)6.2 \text{ per cent of the input.}$$

If, for a rough consideration of the case, we take the heating to be proportional to the total loss, this result would justify us in giving careful consideration to the question of the feasibility of rating up the machine in about the ratio of:

$$6.2 : 7.0.$$

This would bring the rated output up to

$$\frac{7.0}{6.2} \times 160 = 180 \text{ kw.}$$

But before finally determining upon increasing the rating, it would be necessary to examine into the question of the heating of the individual parts, since in decreasing the rotor heating it does not necessarily follow that it is expedient to increase the stator heating to the extent of the amount of the decreased loss in the rotor. On the other hand, in the case of very high speed generators, the heating of the *rotor* conductors will, practically, always constitute the limit, owing to the difficulty of circulating air through a rotor of small diameter and great length. Consequently in the case of very high speed generators the increased rating rendered practicable by the substitution of a low-loss squirrel-cage rotor for a rotor excited with continuous electricity, will often be *much greater* than in the inverse ratio of the respective total losses for the two cases.

The author's present object is, however, to point out that the induction generator has inherent characteristics usually permitting of assigning to it a materially higher rating than that which is appropriate when the same frame is employed in the construction of an induction motor.

Since the slip is 1.2 per cent, the full-load speed is

$$(1.012 \times 500 =) \ 506 \text{ r.p.m.}$$

The Derivation of a Design for an Induction Generator from a Design for a Synchronous Generator. Let us now evolve an induction generator from the 2500-kva. synchronous generator for which the calculations have been carried through in Chapter II.

This machine was designed for the supply of 25-cycle electricity. It had 8 poles and operated at a speed of 375 r.p.m. Let us first plan not to alter the stator in any respect except to employ a nearly-closed slot of the dimensions indicated in Fig. 113. This alteration from the slot proportions employed in the synchronous generator is necessary in order to avoid parasitic losses when the machine is loaded. The necessity arises from the circumstance that, unlike a **synchronous** generator, an **induction** generator must be designed with a very small air-gap. Otherwise it would have an undesirably-low power-factor and would require the supply of too considerable a magnetizing current from the synchronous apparatus with which it operates in parallel. In the case with which we are dealing, we may employ an air-gap depth of only 2 mm. Thus we have:

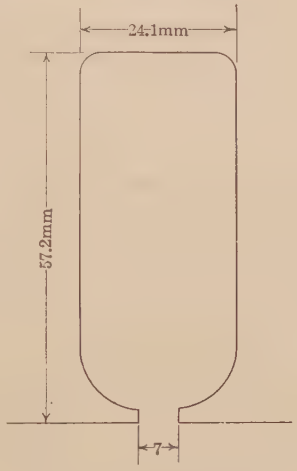


FIG. 113.—Stator Slot for Induction Generator.

$$\Delta = 0.20.$$

The stator has 120 slots. Let us supply the rotor with 106 slots. Each rotor slot may be made 50 mm. deep and 25 mm. wide. The slot is closed, as shown in Fig. 114, by a solid steel wedge with a depth of 10 mm. The conductor is uninsulated and is 40 mm. deep by 25 mm. wide, its cross-section thus being:

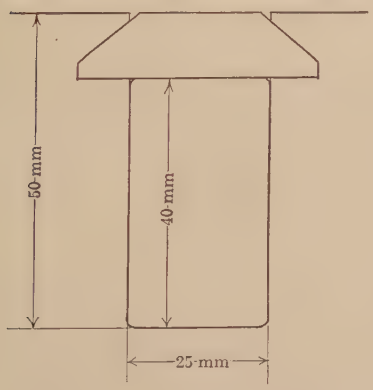


FIG. 114.—Slot for Rotor of Induction Generator.

$$4 \times 2.5 = 10 \text{ sq.cm.}$$

Since there are $(120 \times 10 =) 1200$ stator conductors and $(106 \times 1 =) 106$ rotor conductors, the ratio of transformation is:

$$\frac{1200}{106} = 11.3.$$

The energy component of the full-load current in the stator winding is 120 amperes. Neglecting the magnetizing component of the stator current, we may obtain a rough, but sufficient, approximation to the value of the full-load current per rotor conductor. This is:

$$(11.3\times 120=)1360 \text{ amperes.}$$

The current density in the rotor conductors is thus:

$$\frac{1360}{10}=136 \text{ amp. per sq.cm.}$$

The gross core length is:

$$\lambda g=118 \text{ cm.}$$

Allowing 8 cm. for the projections at each end, the total length of each rotor conductor is

$$(118+2\times 8=)134 \text{ cm.}$$

The aggregate length of the 106 rotor conductors is

$$(106\times 134=)14\,200 \text{ cm.}$$

The corresponding resistance, at 60° Cent. is:

$$\frac{14\,200\times 0.0000020}{10}=0.00284 \text{ ohm.}$$

The I^2R loss in the rotor face conductors at full load is:

$$1360^2\times 0.00284=5250 \text{ watts.}$$

At full load, the current in each end ring is (see p.

$$\frac{106}{\pi\times 8}\times 1360=5700 \text{ amp.}$$

Let us give each end ring a cross-section of 40 sq.cm.

The mean diameter of an end ring must be a little less than D, i.e., a little less than 178 cm. Let us take the mean diameter of the end ring as 165 cm.

The resistance of a conductor equal to the aggregate of the developed length of the two end rings is, at 60° Cent.:

$$\frac{2\times\pi\times165\times0.0000020}{40}=0.000052 \text{ ohm.}$$

The full-load loss in the two end rings is:

$$5700^2\times0.000052=1680 \text{ watts.}$$

The total loss in the squirrel cage, at full load is:

$$5250+1680=6930 \text{ watts.}$$

If our induction generator were to be rated at 2500 kw., this rotor loss would be:

$$\frac{6930\times100}{2\,500\,000}=0.28 \text{ per cent.}$$

We have seen (p. 90) that the loss in the rotor of our 2500 kva. synchronous generator is (at a power-factor of 0.90 and consequently an output of 2250 kw.) 15 500 watts.

Thus the efficiency is considerably greater in the case of the induction generator rating of 2500 kw. Indeed, since the hottest part of the synchronous generator is its rotor, we can easily rate up the machine, when re-modelled as an induction generator, to 3000 kw, the slip then being:

$$\left(\frac{3000}{2500}\times0.28=\right)0.33 \text{ per cent.}$$

Since, however, the air-gap is now so small as to be of but little service as a channel through which to circulate cooling air, the preferable design for the induction generator would consist in a modification in which, instead of employing numerous vertical

ventilating ducts, the air is circulated through 120 longitudinal channels, one just below each stator slot. Two other methods of air circulation which have been employed on the Continent

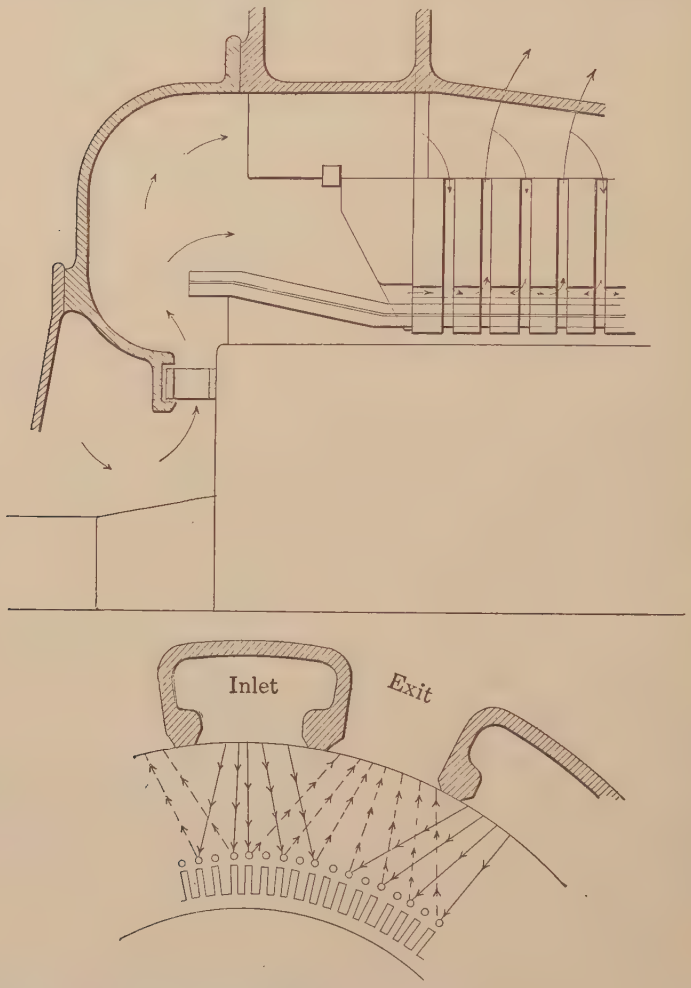


FIG. 115.—A Method of Ventilation Suitable for an Induction Generator.

of Europe for synchronous generators and are especially appropriate for induction generators, are shown in Figs. 115 and 116. In that indicated in Fig. 115, the air from the fans on the ends of the rotor is passed to a chamber at the external surface of the

armature core. This chamber opens into air ducts in a plane at right angles to the shaft. Suitably shaped space blocks lead the air, in a tangential direction, to axial ducts just back of the stator slots. The air then flows axially through one section, to the next air duct, and then outwardly in a tangential direction to a chamber at the outer surface of the core. This chamber is adjacent to the one first mentioned and leads to the exit from the stator frame. Looking along the axis of the shaft, the air flows in a V-shaped path, the axial duct back of the stator slots

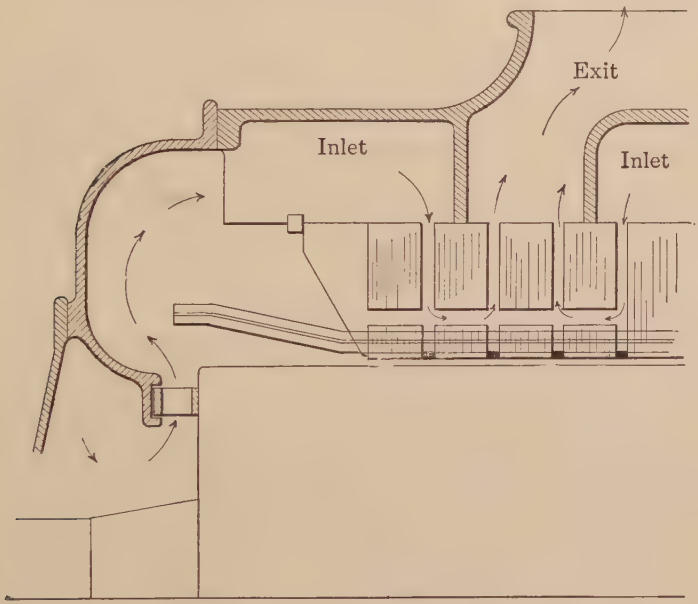


FIG. 116.—An Alternative Method of Ventilation Suitable for an Induction Generator.

being at the apex. Thus the two legs of the V are separated axially by a single armature section.

The other method (Fig. 116) which is also independent of the radial depth of the air-gap, consists in dividing the stator frame into cylindrical chambers placed side by side. The air is forced into a chamber from which it first passes radially toward the shaft, then axially to adjacent air ducts, and finally outwardly to a chamber alongside the one first mentioned. This last chamber communicates with the outer air.

Since the loss in the squirrel cage is 0.33 of 1 per cent, in the case of this induction generator, the speed at rated load will be $1.0033 \times 375 = 376.2$ r.p.m.

For the 2-mm. air-gap which we are now employing, the mmf. calculations for the phase pressure of 6950 volts, may (without attempting to arrive at a needlessly exact result) be estimated as follows:

Air-gap	1300 ats.
Teeth	700 "
Stator core	300 "
Rotor core	300 "
Total mmf. per pole	= 2600 ats.

Thus each phase must supply 1300 ats. There are 25 turns per pole per phase. Thus the no-load magnetizing current is:

$$\frac{1300}{25 \times \sqrt{2}} = 37 \text{ amperes.}$$

Without going into the estimation of the circle factor it is evident that the wattless component for full load of 3000. kw. will be a matter of some 45 amperes.

The current output at full load of 3000 kw. is:

$$\frac{3\,000\,000}{3 \times 6950} = 144 \text{ amperes.}$$

The total current in each stator winding) at full load is:

$$\sqrt{45^2 + 144^2} = 151 \text{ amperes.}$$

The power-factor at full load is:

$$\frac{144}{151} = 0.95.$$

It must be remembered that this design is for the moderate speed of 375 r.p.m. For high-speed designs (say 1500 r.p.m.

at 25 cycles) the power-factor, in large sizes, may readily be brought up to 0.97.

In the paper to which reference has already been made (Hobart and Knowlton's "The Squirrel-cage Induction Generator"), mention is made of a comparative study which has been carried out for two 60-cycle, 3600 r.p.m. designs, one for a 2500-kw. synchronous generator and the other for a 2500-kw. induction generator, both for supplying a system at unity power-factor. The leading data of these two designs were as follows:

	Synchronous Generator.	Induction Generator.
Core loss	30 kw.	30 kw.
Primary I^2R loss	9.5 "	10.5 "
Secondary I^2R loss	6.5 "	2.8 "
Windage	35 "	35 "
Total loss, excluding friction of bearings	81 "	78.3 "
Efficiency, excluding friction of bearings	96.9 per cent	97 per cent
Per cent slip at rated load	0 "	0.11 "

The necessity for employing in induction generators a very small air-gap renders it more important than in synchronous generators that the magnetomotove force per slot (expressed in ampere-conductors at rated load) shall be small. This requires (in the case of large machines), the subdivision of the winding amongst a larger number of slots per pole than would be sufficient for a synchronous machine of the same rating. The subdivision of the winding in many slots increases the desirability of employing a low pressure and renders it especially advantageous to adopt the plan of stepping-up to the line pressure through a compensator. The most economical ratio of transformation for the compensator is 2 : 1. Consequently if a large induction generator is to supply a 12,000-volt system, it should be wound for 6000 volts and should supply the system through a 2 : 1 compensator.

In large sizes of high-speed induction generators, it may be preferable to use a rotor in which the shaft and core are cut from a solid piece. Such a construction is not objectionable for the rotor of an induction generator, for we can design

it with a slip at rated load, of, say, two-tenths of 1 per cent. This corresponds to a rotor periodicity of only $(0.002 \times 60 =) 0.12$ of a cycle per second, or 7.2 cycles per minute, and under such conditions there is no necessity for employing a laminated core.

On account of the small air-gap of induction generators, the value of the critical speed of vibration is especially important. If possible, the critical speed should be at least 10 per cent above normal. If other important reasons require employing a critical speed below normal, it should be **considerably** below, care being taken that the second critical speed is also removed from the normal, preferably above it. With such a design the rotor should have a very careful running balance before it is placed in the machine. A damping bearing could be used to prevent the rubbing of the rotor and stator if for any reason the machine should be subjected to abnormal vibration.

It should be noted that in cases where the critical speed must be below the normal speed, the air-gap cannot be so small as would be preferred from the standpoint of minimizing the magnetizing current. A consideration tending to the use of a shaft with a critical speed below the normal running speed relates to the lower peripheral speed thereby obtained at the bearings.

CHAPTER VIII

EXAMPLES FOR PRACTICE IN DESIGNING POLYPHASE GENERATORS AND MOTORS

In connection with courses of lectures on the subject-matter of this treatise, the author has had occasion to set a number of examination papers. In the present chapter some of these examination papers are reproduced and it is believed that they will be of service in acquiring ability to apply the designing principles discussed in the course of the preceding chapters.

Paper Number I

1. The leading data of a 12-pole, 250-r.p.m., 25-cycle, 11,000-volt, 3000-kva., Y-connected, three-phase generator are as follows:

Output in kilovolt-amperes	3000
Number of poles	12
Terminal pressure	11 000 volts
Style of connection of stator windings	Y
Current per terminal	157 amperes
Speed in r.p.m.	250
Frequency in cycles per second	25
Gross core length of armature (λ_g)	100 cm.
Total number of slots	108
Conductors per slot	10
(All conductors per phase are in series).	

The no-load saturation curve is given in Fig. 117.

What is the armature strength in ampere-turns per pole when the output is 157 amperes per phase? What, at 25 cycles, is the reactance of the armature winding in ohms phase? Plot

saturation curves when the generator is delivering full load current (i.e., 157 amperes) for:

I. Power-factor = 1.00
II. " = 0.70
III. " = 0.20

For a constant excitation of 11 250 ampere-turns per field spool, what will be the percentage drop in terminal pressure when

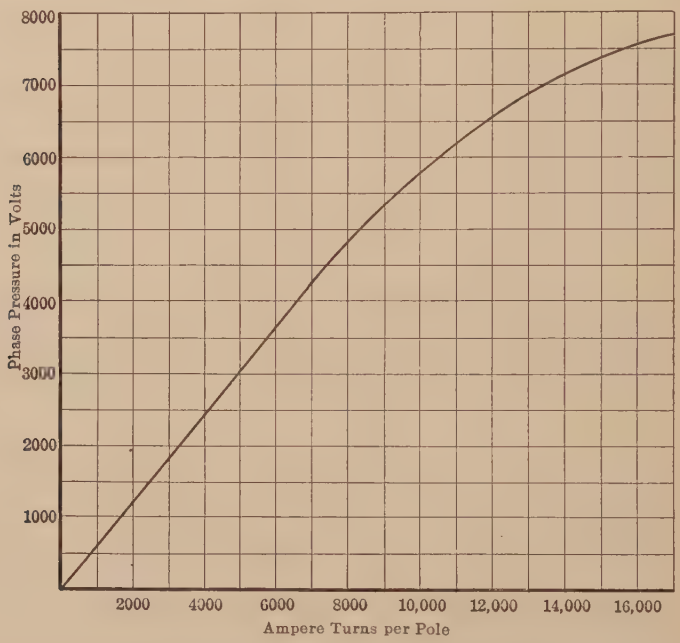


FIG. 117.—No-load Saturation Curve for the 3000-kva. Three-phase Generator Described in Paper No. 1.

the output is increased from 0 amperes to full-load amperes, i.e., to 157 amperes:

I. At power-factor = 1.00
II. " = 0.70
III. " = 0.20

For a constant terminal pressure of 11 000 volts, what will be the required increase in excitation in going from 0 amperes output to full load (i.e., 157) amperes output

I. At power-factor = 1.00
II. " = 0.70
III. " = 0.20

In calculating theta (θ) for this generator, work from the data given on pp. 45 and 48.

2. A 100-h.p., 12-pole, 500-r.p.m., 50-cycle, 500-volt, Y-connected, three-phase squirrel-cage induction motor has a no-load current of 23 amperes and a circle ratio (σ) of 0.058. The core loss is 2200 watts, and the friction loss is 1400 watts. At rated load the I^2R losses are

Stator . . . . . 2850 watts
Rotor . . . . . 2850 "

[By means of a circle diagram and rough calculations based on the above data, plot the efficiency, the power-factor, and the amperes input, all as functions of the output, from no load up to 100 per cent overload.

Paper Number II

1. In a certain three-phase, squirrel-cage induction motor, the current per phase at rated load is 60 amperes. The no-load current is 20 amperes. The circle ratio (σ) is 0.040. Construct the circle diagram of this motor. What is its power-factor at its rated load? What is the current input per phase at the load corresponding to the point of maximum power-factor, and what is the maximum power-factor? If the "stand-still" current is 500 amperes (i.e., if the current when the full pressure is switched on to the motor when it is at rest, is 500 amperes), ascertain graphically by the aid of the circle diagram the power-factor at the moment of starting. If the stator windings are Y-connected and if the terminal pressure is 1000 volts (the pressure per phase consequently being $\frac{1000}{\sqrt{3}} = 577$ volts), what would be the input

to the motor, in watts, at the moment of starting under these conditions? Describe, without attempting to give *quantitative* data, the means usually employed in practice to start such a motor with much less than the above large amount of power and with much less current than the " stand-still " current given above.

2. A three-phase, Y-connected, squirrel-cage induction motor has 48 stator slots and 12 conductors per slot. The terminal pressure is 250 volts (the pressure per phase consequently being 144 volts). The motor has 4 poles and its speed, at no load, is 1500 r.p.m. Estimate the magnetic flux per pole. The squirrel cage comprises 37 face conductors, each having a cross-section of 0.63 sq.cm. and each having a length between end rings of 20 cm. Each end ring has a cross-section of 2.45 sq.cm. and a mean diameter of 20 cm. Calculate the I^2R loss in the squirrel cage when the current in the stator winding is 17.3 amperes. If, without making any further alteration in the motor, the cross-section of the end rings is reduced to one-half, what will be the I^2R loss in the squirrel cage for this same current? What general effect will this change have on the " slip " ? On the starting torque? On the efficiency? On the heating?

Paper Number III

Determine appropriate leading dimensions and calculate as much as practicable of the following design:

A 25-cycle, 250-volt, three-phase, squirrel-cage induction motor for 750 h.p. and a synchronous speed of 250 r.p.m.

Calculate as much of this design as time permits. If, however, you do not get on well with the entire design, then take some particular part of the design, say—the estimation of the magneto-motive force and stator winding and carry it out in detail.

While there is still time, bring together the leading dimensions and properties in a concise schedule. Make use of the printed specifications (to be had on request) if desired.

Paper Number IV

A no-load saturation curve is given in Fig. 118. This applies to a three-phase alternator with a rated output of 850 kw. at

2880 volts per phase and 94 r.p.m. 32 pole, 25-cycle and unity power-factor. At rated load the reactance voltage is 945 volts per phase, and the resultant maximum armature strength is 3900 ampere-turns per pole.

Estimate the inherent regulation of this machine for rated full-load current of 98.5 amperes at unity power-factor. Also for this same current, but at a power-factor of 0.8.

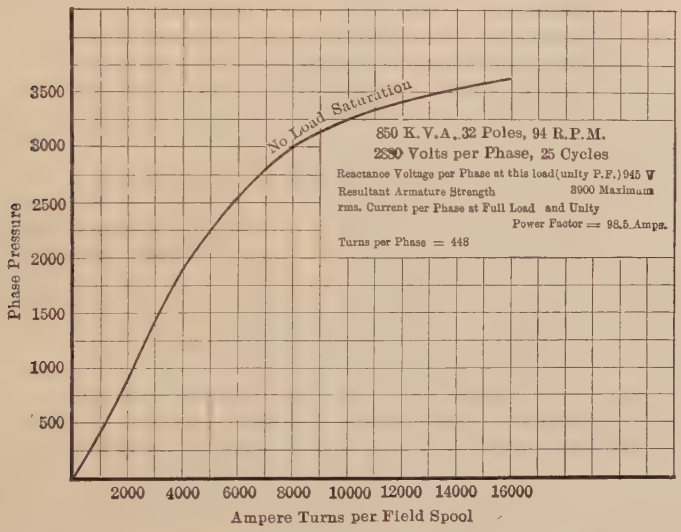

No Load Saturation
850 K.V.A., 32 Poles, 94 R.P.M.
2880 Volts per Phase, 25 Cycles
Reactance Voltage per Phase at this load (unity P.F.) 945 V
Resultant Armature Strength 3900 Maximum
rms. Current per Phase at Full Load and Unity Power Factor = 98.5 Amps.
Turns per Phase = 448
Phase Pressure
3500
3000
2500
2000
1500
1000
500
2000 4000 6000 8000 10000 12000 14000 16000
Ampere Turns per Field Spool

FIG. 118.—No-load Saturation Curve for the 850-kva. Three-phase Generator Described in Paper No. IV.

Draw a group of saturation curves (as indicated roughly in Fig. 119, for 98.5 amperes and for power-factors of 1.0, 0.8 and 0.

Plot excitation regulation curves of this machine for normal pressure of 2880 volts per phase, and for power-factors of 1.0, 0.8 and 0.

PAPER NUMBER V

Answer only *one* of the two following questions:

(Do not necessarily attempt to do more than arrive at the approximate general outline for the machine. If, afterwards, you have time, work out and tabulate its leading properties.

But the chief consideration is that you should demonstrate your ability to make a rough estimate of the most probably correct design.)

1. *Design* a 4-pole, Y-connected, 30-cycle, three-phase, squirrel-cage induction motor for a primary terminal pressure of 1000 volts (577 volts per phase) and for a rated output of 100 h.p.

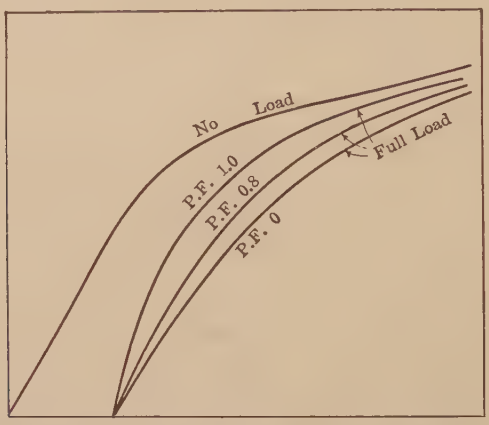


Fig. 119.—Rough Indication of the Saturation Curves Called for in Paper No. IV.

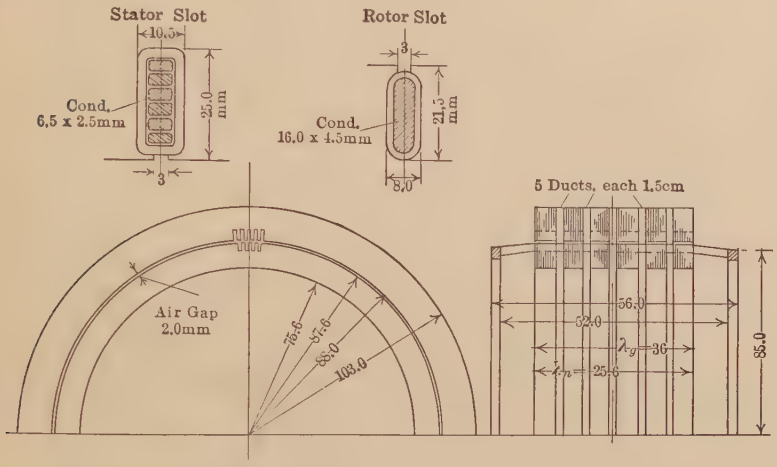
2. *Design* a 50-cycle, Y-connected polyphase generator for a rated output of 1500 kva. at a speed of 375 r.p.m. and for a terminal pressure of 5000 volts (2880 volts per phase).

Paper Number VI

For the induction motor shown in Fig. 120:

1. Make a rough estimate of a reasonable normal output to assign to the motor.
2. Estimate the no-load current.
3. Estimate the circle ratio.
4. Estimate the breakdown factor at the output you have assigned to the motor.
5. Estimate the losses and efficiency at the output you have assigned to the motor.
6. Estimate the temperature rise at the output you have assigned to the motor.
7. Estimate the power-factor at various loads.

(NOTE.—If, rightly or wrongly, you consider that some essential data have not been included in Fig. 120, do not lose time over the matter, but make some rational assumption—stating on your paper that you have done so—and then proceed with the work.)



THREE-PHASE SQUIRREL-CAGE INDUCTION MOTOR.

Number of poles	12
Terminal pressure	1000 volts
(∴ Pressure per phase $= \frac{1000}{\sqrt{3}} =$	578 volts
Synchronous speed in r.p.m.	500
Stator connections	Y

	Stator.	Rotor.
Number of slots	180	216
Dimensions of slot (depth × width)	25 × 10.5 mm.	21.5 × 8.0 mm.
Slot opening	3 mm.	3 mm.
Conductors per slot	6	1
Dimensions of bare conductor	2.5 × 6.5 mm.	4.5 × 16.0 mm.
Number of end rings		2
Section of end ring		20 × 20 mm.

Dimensions in centimeters and millimeters.

FIG. 120.—Sketches and Data of Induction Motor of Paper No. VI.

PAPER NUMBER VII

Design the following three-phase, squirrel-cage induction motor:

Rated output = 30 h.p.;
Synchronous speed = 1000 r.p.m.;
Periodicity = 50 cycles per second;
Pressure between terminals = 500 volts;
Y-connected stator winding.

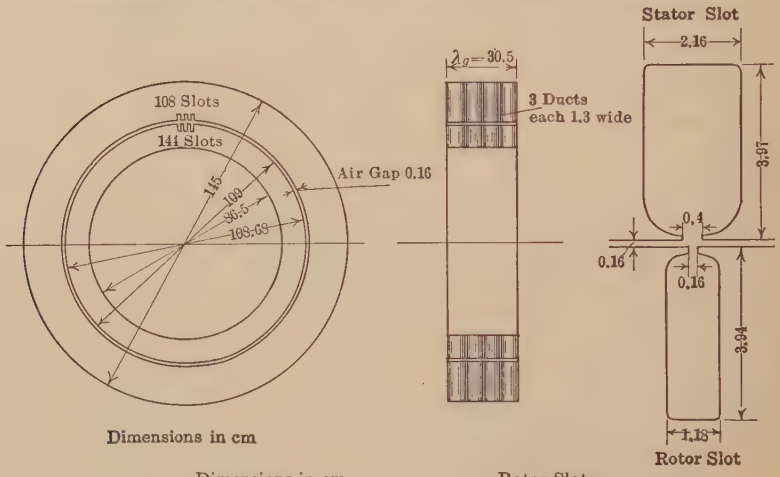
$\left(\therefore \text{ Pressure per phase} = \frac{500}{\sqrt{3}} = 288 \text{ volts.}\right)$

Proportion the squirrel-cage rotor for 4 per cent slip at rated load. Carry the design as far as time permits, but devote the last half hour to preparing an orderly table of your results.

NOTE.—If, rightly or wrongly, you conclude that some essential data have not been included in the above, do not lose time over the matter, but make some rational assumption—stating on your paper that you have done so—and then proceed with the work.

PAPER NUMBER VIII

For the induction motor design of which data is given in Fig. 121, estimate the losses at rated load. Draw its circle diagram.



Dimensions in cm.	Rotor Slot.
Output in H.P.	220
Number of poles	12
Connection of stator windings	Y
Periodicity in cycles per second	50
Volts between terminals	5000
Stator Winding.	
Total number of stator conductors	3564
Number of stator conductors per slot	33, each consisting of two components
Dimensions of stator conductor (bare)	2 in parallel, each 2.34 mm. diameter.
Mean length of stator turn	159 cm.
Rotor Windings.	
Total number of rotor conductors	288
Number of rotor conductors per slot	2
Number of phases in rotor winding	3
Dimensions of rotor conductor (bare)	16×9 mm.
Mean length of rotor turn	150 cm.

FIG. 121.—Sketches and Data of the 220-H.P. Induction Motor of Paper No. VIII.

Plot its efficiency, power-factor and output, using amperes input as abscissae, from no load up to the breakdown load. Estimate

the starting torque and the current input to the motor at starting, when half the normal voltage is applied to the terminals of the motor.

NOTE.—If you are of opinion that sufficient data have not been given you to enable all the questions to be answered, do not hesitate to make some reasonable assumption for such missing data.

PAPER NUMBER IX

Design a 300-h.p., three-phase, 40-cycle, 240-r.p.m., squirrel-cage induction motor for a terminal pressure of 2000 volts. Let the stator be Y-connected. Obtain γ, the ratio of the no-load to the full-load current, σ, the circle ratio, and bdf., the breakdown factor; and further data if time permits. Employ the last half hour in criticising your own design, and in stating the changes you would make with a view to improving it, if you had time.

PAPER NUMBER X

During the entire six hours at your disposal, design a three-phase squirrel-cage induction motor, to the following specification:

Normal output in h.p.	150
Periodicity in cycles per second	50
Synchronous speed (i.e. speed at no load), in r.p.m.	250
Terminal pressure in volts	400
Connection of phases	Y
Pressure per phase in volts	231

The last two hours, i.e., from 3 to 5 P.M., must be devoted to tabulating the results at which you have arrived, and to the preparation of outline sketches with principal dimensions. As to the electrical design, the following particulars will be expected to be worked out or estimated:

1. Ratio of no-load to full-load current, (γ).
2. Circle ratio, (σ).
3. Circle diagram to scale.
4. Breakdown factor (bdf.).
5. Per cent slip at rated load.
6. Losses in stator winding, at rated load.

7. Losses in rotor winding (i.e., squirrel-cage losses at rated load).
8. Core losses.
9. Friction.
10. Efficiency at rated load.
11. Power-factor at rated load.
12. Estimate of thermometrically determined ultimate temperature rise at rated load.

(NOTE.—The students are permitted to bring in any books and notes and drawing instruments they wish. They are also permitted to fill out and to hand in as a portion of their papers, specification forms which they have prepared in advance of coming to the examination.)

PAPER NUMBER XI

Any notes or note-books and other books may be used, but students are put on their honor not to discuss any part of their work in the lunch hour.

A three-phase, squirrel-cage induction motor complies with the following general specifications:

Rated load in h.p.	= 90
Periodicity in cycles per second	= 40
Speed in r.p.m. at synchronism	=800
Terminal pressure	=750
Connection of phases	=Y

(1) Given the following data, estimate γ, the ratio of no-load to full-load current:

Average air-gap density	=3610 lines per sq.cm.
Air-gap depth	=0.91 mm.
Air-gap mmf. ÷ total mmf.	=0.80
Total number of conductor on stator	=576.

(2) Given the following data, estimate σ, the circle ratio:

Internal diameter of stator laminations	=483 mm.
Slot pitch of stator at air-gap	=21.1 mm.
Slot pitch of rotor at air-gap	=24.8 mm.
Gross core length; λg	=430 mm.

(3) Draw the circle diagram. Determine the primary current:

(*a*) At point of maximum power-factor.
(*b*) breakdown load.

(4) Plot curves between:

(*a*) H.p. and η, the efficiency.
(*b*) H.p. and G, the power-factor.
(*c*) H.p. and slip.
(*d*) H.p. and amperes input.

Given the following data:

Section of stator conductor = 0.167 sq.cm.
Dimensions of rotor conductor = 1.27 cm. × 0.76 cm.
(One rotor conductor per slot)
Length of each rotor conductor = 49.5 cm.
Diameter of end rings (external) = 45 cm.
Number of end rings at each end = 2, each 2.54 cm. × 0.63 cm.
Total constant losses = 2380 watts.

(5) Calculate the starting current and torque when compensators supplying 33, 40 and 60 per cent of the terminal pressure are used.

(6) Calculate new end rings so that with a 2 : 1 compensator the starting torque shall be equal to one-half torque at rated load.

(7) Designating the first motor as A and the second one (i.e., the modification obtained from Question 6) as B, tabulate the component losses at full load in two parallel (vertical columns).

Estimate the watts total loss per ton weight of motor (exclusive of side-rails and pul'ey).

(8) If the squirrel-cage of the first motor A is replaced by a three-phase winding having the same equivalent losses at full load, calculate the resistance per phase which would be required, external to the slip rings, in order to limit the starting current to 70 amperes. What would be the starting torque, expressed as percentage of full-load torque, with this external resistance inserted?

Paper Number XII

For the three-phase induction motor of which data is given below, make calculations enabling you to plot curves with amperes input per phase as abscissae and power-factor, efficiency and output in h.p. as ordinates.

Rated load	= 60 h.p.
Periodicity in cycles per second	= 50
Speed at synchronism	= 600 r.p.m.
Terminal pressure	= 550 volts
Connection of phases	= Y
Average air-gap density	= 3900 lines per sq.cm.
Air-gap depth	= 0.9 mm.
Total mmf. ÷ air-gap mmf	= 1.2
Total no. of conductors on stator	= 720
Circle ratio	= 0.061
I^2R losses at rated load	= 2840
Constant losses	= 2050

Paper Number XIII

The data given below are the leading dimensions of the stator and rotor of a 24-pole three-phase induction motor with a Y-connected winding suitable for a 25-cycle circuit. Ascertain approximately by calculation the suitable terminal voltage for this induction motor and give your opinion of the suitable rated output. Proceed, as far as time permits, with the calculation of the circle ratio and of the no-load current, and construct the circle diagram. Abbreviate the calculations as much as possible by reasonable assumptions for the less important steps, thus obtaining more time for the steps where assumptions can less safely be made.

Stator. Dimensions in MM.

External diameter	2800
Internal diameter	2440
Gross core length	385
Net core length	299
Winding	3-phase, Y-connected
Number of slots	216
Depth × width of slot	51 × 21
Conductors per slot	12
Section of conductors, sq.cm	0.29

Rotor.

External diameter	2434.5
Internal diameter	2144
Winding	3-phase, Y-connected
Number of slots	504
Depth × width of slot	35×9.5
Conductors, per slot	2
Section of conductor, sq.cm	0.811

[This paper to be brought in for the afternoon examination (see Paper No. XIV), as certain data in it will be required for the afternoon examination.]

Paper Number XIV

The 24-pole stator which you employed this morning (see Paper No. XIII), for an induction motor, will, if supplied with a suitable internal revolving field with 24 poles, make an excellent three-phase, 25-cycle, Y-connected alternator. What would be an appropriate value for the rated output of this alternator? Without taking the time to calculate it, draw a reasonable no-load saturation curve for this machine. From this curve and from the data of the machine and your assumption as to the appropriate rating, calculate and plot a saturation curve for the rated current when the power-factor of the external circuit is 0.80.

Paper Number XV

1. Of two 50-cycle, 100-h.p., 500-volt, three-phase induction motors, one has 4 poles and the other has 12 poles.

(*a*) Which will have the higher power-factor?
(*b*) " " " efficiency?
(*c*) " " " current at no load?
(*d*) " " " breakdown factor?

Of two 750 r.p.m., 100-h.p., 500-volt, three-phase induction motors, one is designed for 50 cycles, and the other for 25 cycles,

(*e*) Which will have the higher power-factor?
(*f*) " " " current at no load?
(*g*) " " " breakdown factor?

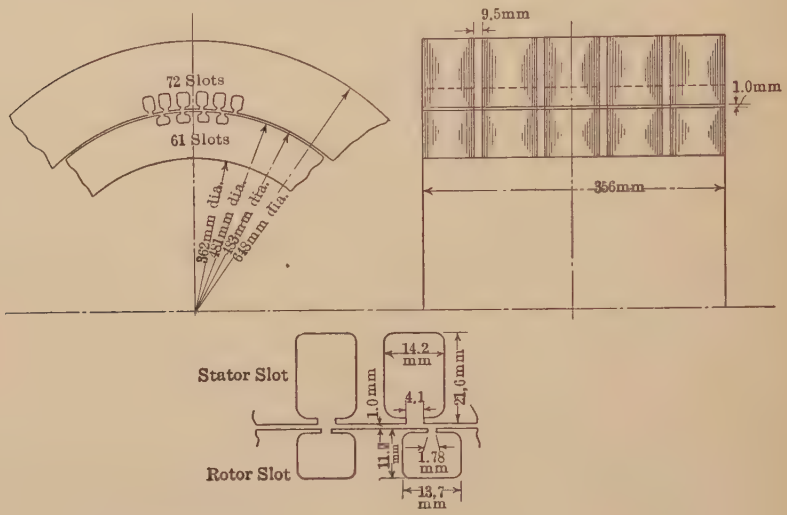
2. Describe how to estimate the temperature rise of an induction motor.

3. Describe how to estimate the magnitude and phase of the starting current of a squirrel-cage induction motor.

Paper Number XVI

Answer one of the following two questions.

Question I. For the three-phase squirrel-cage, induction motor of which data are given in Fig. 122:



Terminal pressure	750
Method of connection	Y
Pressure per phase	432
Speed in r.p.m.	800
Full load primary current input per phase	59
Number of primary conductors per slot	8
Periodicity in cycles per second	40

	Length in cm.	Density in Lines per sq.cm.	Ampere-turns per cm. of Length.	Total Ampere-turns.
Stator teeth				
Stator core				
Rotor teeth				
Rotor core				
Air-gap				
Total mmf. per pole				

Fig. 122.—Sketches and Data of the Three-phase Induction Motor of Question I in Paper No. XVI.

(*a*) Work out the magnetic circuit calculations required to fill in the table indicated, and enter up the results in a similar table.

(*b*) Obtain γ and σ for the motor.

Question II. For a certain three-phase, slip ring, induction motor the following data apply:

Periodicity in cycles per sec	50
Speed at no load in r.p.m.	500
γ (ratio of no-load to full-load current)	0.36
σ (circle ratio)	0.0742
Rated output in h.p.	300
Terminal pressure	700
Pressure per phase	405
Connection of phases	Y
Stator resistance per phase (ohm)	0.030
Rotor resistance per phase (ohm)	0.022
Total core loss (watt)	4080
Friction and windage loss (watt)	2000
Ratio number of stator to number of rotor conductors	1.28

(*a*) Draw the circle diagram.

(*b*) Plot curves of efficiency, power-factor, output in horse-power and slip, all as a function of the current input per phase.

Paper Number XVII

1. For an armature having an air-gap diameter, $D=65$ cms. and a gross core length $\lambda g=35$ cms.

What would be a suitable rating for a 500-volt, 500-r.p.m. machine of these data:

1st. As a 25-cycle induction motor.

2d. As a 25-cycle alternator.

Select one of these cases and work out the general lines of the design as far as time permits.

2. In Fig. 123 are given data of the design of an alternator for the following rating:

2500 kva., 3-phase, 25-cycle, 75 r.p.m., 6500-volts, Y-connected. The field excitations for normal voltage of 2200 volts and for 1.2 times normal voltage (2640 volts), are given. From these values the no-load saturation curve may be drawn.

Estimate (showing all the necessary steps in the calculations).

(*a*) The field ampere-turns required for full terminal voltage at full-load kva. at power-factors 1.0 and 0.8; and the pressure regulation for both these cases.

(*b*) The short circuit current for normal speed and with no-load field excitation for normal voltage.

(*c*) The losses and efficiency at rated full load and $\frac{1}{2}$ load at 0.8 power-factor. Also the armature heating.

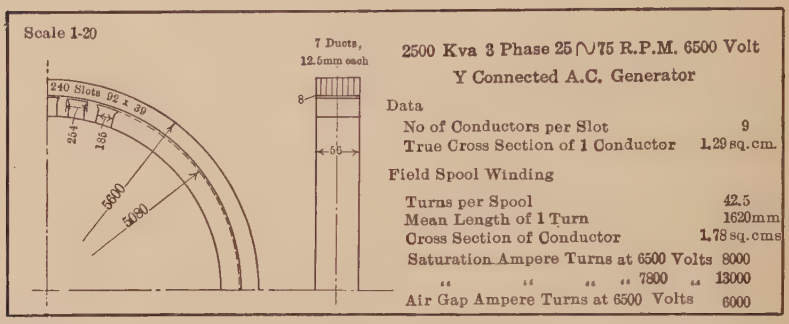

Scale 1-20
7 Ducts, 12.5mm each
240 Slots 92 x 39
234
198
5600
5080
8
56
2500 Kva 3 Phase 25∩∪75 R.P.M. 6500 Volt
Y Connected A.C. Generator
Data
No of Conductors per Slot 9
True Cross Section of 1 Conductor 1.29 sq.cm.
Field Spool Winding
Turns per Spool 42.5
Mean Length of 1 Turn 1620mm
Cross Section of Conductor 1.78 sq.cms
Saturation Ampere Turns at 6500 Volts 8000
" " " " 7800 " 13000
Air Gap Ampere Turns at 6500 Volts 6000

FIG. 123.—Data of the Design of the 2500-kva. Alternator of Question 2 of Paper No. XVII.

3. In Fig. 124 are given data of the design of an induction motor for the following rating:

40-h.p., 600-r.p.m., 50-cycles, 500-volt Δ-connected, three-phase induction motor.

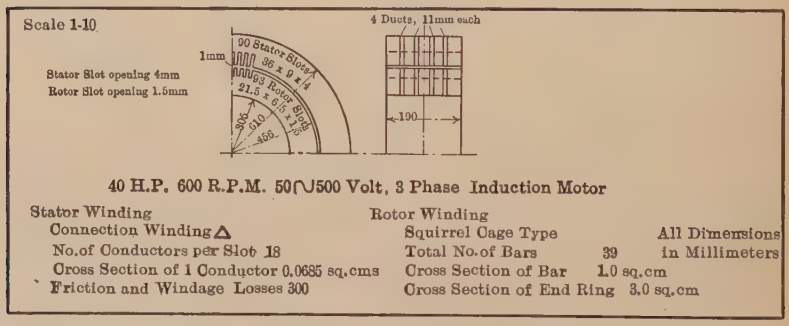

Scale 1-10
4 Ducts, 11mm each
1mm
90 Stator Slots
36 x 9 x ¼
93 Rotor Slots
21.5 x 6.5 x 1.5
Stator Slot opening 4mm
Rotor Slot opening 1.5mm
190
40 H.P. 600 R.P.M. 50∩∪500 Volt, 3 Phase Induction Motor
Stator Winding
Connection Winding Δ
No.of Conductors per Slot 18
Cross Section of 1 Conductor 0.0685 sq.cms
Friction and Windage Losses 300
Rotor Winding
Squirrel Cage Type
Total No.of Bars 39
Cross Section of Bar 1.0 sq.cm
Cross Section of End Ring 3.0 sq.cm
All Dimensions in Millimeters

FIG. 124.—Data of the Design of the 40-H.P. Motor of Question 3 of Paper No. XVII.

Estimate (showing all the necessary steps in the calculations).

(*a*) Circle ratio (σ);

(*b*) No-load current in per cent of full-load current;

(*c*) Breakdown factor ;

(*d*) Maximum power-factor.

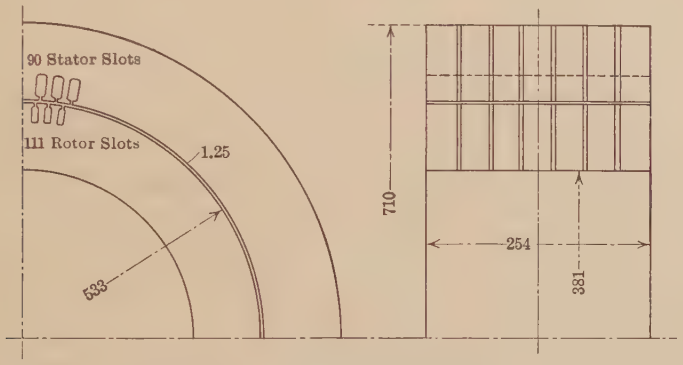
(*e*) Losses and efficiencies and heating at $\frac{1}{2}$ load and at rated full load.

(*f*) Slip at full load.

Paper Number XVIII

(*You may use notes, curves or any other aids.*)

1. The leading particulars of a certain induction motor are given in Fig. 125.



80. H.P., 600 r.p.m., 50 Cycles, 500 Volts, Δ-Connected Squirrel-cage Induction Motor

Data:

Number of stator slots		90
Slot dimensions	Depth	36.0
	Width	11.0
	Opening	6.0
Number of rotor slots		111
Slot dimensions	Depth	21.5
	Width	6.5
	Opening	1.5

Windings:

Stator		
	Conductors per slot	12
	Cross-section of conductor	0.138 sq.cm.
Rotor:		
	Bars per slot	1
	Cross-section of bar	1.0 sq.cm.
	Cross-section of end ring	3.6 sq.cm.

All dimensions in millimeters.

Fig. 125.—Sketches and Data of the 80-H.P. Induction Motor of Question 1 of Paper No. XVIII.

(*a*) Estimate γ the ratio of the no-load current to the current at normal rating.

(*b*) Estimate σ the circle ratio, and also estimate the maximum power-factor.

(*c*) Estimate bdf., the breakdown factor.

(*d*) Estimate the copper losses at normal rating.

(*e*) Estimate the core losses at normal rating.

(*f*) Estimate the watts per square decimeter of equivalent gap surface.

(*g*) Estimate the efficiency at normal rating.

(*h*) Estimate the power-factor at normal rating.

(*i*) Estimate the T.W.C. (the total works cost).

2. (*a*) Deduce the leading proportions for a three-phase alternator for a normal rating of 1500 kw., 1000 r.p.m., 50 cycles, 6 poles, 11 000 volts. Do not go into detail, but go far enough to give an opinion as to the suitable values for D, λg, number of slots, conductors per slot, and field excitation at no load. State briefly your reasons for choosing each of these values.

(*b*) What, in a general way, should be the changes in the general order of magnitude of these quantities for a design for the same rated output and voltage at

(1) 50 cycles, 250 r.p.m., 24 poles,

and the changes necessary in this second design in order to obtain a design for

(2) 25 cycles, 250 r.p.m., 12 poles.

Paper Number XIX

Work up a rough outline for a design for a 25-cycle, three-phase generator to supply at a pressure of 10 000 volts (Y-connected with 5770 volts per phase). The generator is to have a rated capacity of 3000 kva. at a power-factor of 0.90, and is to be run at a speed of 125 r.p.m. Design the machine to give good regulation of the pressure, and work out the inherent regulation at various power-factors. Work out any other data for which you find time, and devote the last half hour to an orderly tabulation of your results.

Paper Number XX

Work up a rough outline for a design for a 25-cycle, 8-pole, three-phase, squirrel-cage induction motor for a rated output of 80 h.p.

The terminal pressure is 600 volts, i.e., 346 volts per phase,

and the stator windings should be Y-connected. The load will not be thrown on the motor until it is up to speed.

During the last half hour prepare an orderly tabulation of the leading results which you have found time to work out.

Paper Number XXI

Commence the design of a three-phase, 100-h.p., squirrel-cage induction motor with a Y-connected stator for a synchronous speed of 375 r.p.m. when operated from a 25-cycle circuit with a line pressure of 500 volts. (The pressure per stator winding is consequently $\frac{500}{\sqrt{3}} = 288$ volts.) Try and carry the design as far as determining upon the gap diameter and the gross core length, the number of stator slots, the number of stator conductors per slot, the flux per pole and the external diameter of the stator laminations and the internal diameter of the rotor laminations. Then tabulate these data in an orderly manner before proceeding further. Then, if time permits, make the magnetic circuit calculations and estimate the magnetizing current.

Paper Number XXII

(Answer one of the following two questions.)

1. Which would be the least desirable, as regards interfering with the pressure on a 50-cycle net work, low-speed or high-speed induction motors? Why?

Of two 1000-volt, 100-h.p., 750-r.p.m., three-phase induction motors, which would have the highest capacity for temporarily carrying heavy overloads, a 25-cycle or a 50-cycle design? Which would have the highest power-factor? Which the lowest current when running unloaded?

Of two 1000-volt, 100-h.p., three-phase induction motors for 25 cycles, one is for a synchronous speed of 750 r.p.m. and the other is for a synchronous speed of 150 r.p.m. Assuming rational design in both cases, which has the higher power-factor? Which the lower current when running light? Which the higher breakdown factor? Which the higher efficiency? Three 100-

h.p., 1000-volt designs have been discussed above. Their speeds and periodicities may be tabulated as follows:

Designation.	Synchronous speed in r.p.m.	Periodicity in cycles per sec.
A	750	25
B	750	50
C	150	25

Assume that these are provided with low-resistance squirrel-cage windings. Make rough estimations of the watts total loss per ton of total weight of motor for each case.

2. A three-phase, Y-connected, squirrel-cage induction motor has the following constants:

External diameter stator core	1150 mm.
Air-gap diameter (D)	752 mm.
Internal diameter rotor core	324 mm.
Diameter at bottom of stator slots	841 mm.
Diameter at bottom of rotor slots	710 mm.
No. of stator slots	72
No. of rotor slots	89
Conductors per stator slot	5
Conductors per rotor slot	1
Width of stator slot	19 mm.
Width of rotor slot	12 mm.
Space factor stator slot	0.51
Space factor rotor slot	0.80
Gross core length (λg)	190 mm.
Net core length (λn)	148 mm.
Polar pitch (τ)	394 mm.
Depth of air-gap (Δ)	1.5 mm.
Peripheral speed	38.6 mps.
Output coefficient (ξ)	2.37

The pressure per phase is 1155 volts, the pressure between terminals being 2000 volts.

What is the rated output of the motor in h.p.?

What is the speed in r.p.m.?

What is the periodicity in cycles per second?

What is the flux per pole in megalines?

What is the stator I^2R loss?

What is the stator core loss?

What cross-section must be given to the copper end rings in order that the slip at rated load shall be 2 per cent?

Estimate the efficiency at $\frac{1}{4}$, $\frac{1}{2}$ and full load.

Estimate σ the circle ratio.

Estimate γ the ratio of the magnetizing current to the current at rated load.

What is the power-factor at $\frac{1}{4}$, $\frac{1}{2}$ and full load?

What is the breakdown factor?

Estimate the probable temperature rise after continuous running at rated load.

Remember that some of these questions are most readily solved by constructing a circle diagram and combining the dimensions scaled off from it, with slide-rule calculations.

APPENDIX I

A BIBLIOGRAPHY OF I. E. E. AND A. I. E. E. PAPERS ON THE SUBJECT OF POLYPHASE GENERATORS.

1891.

M. I. Pupin.—On Polyphase Generators. (Trans. Am. Inst. Elec. Engrs., Vol. 8, p. 562).

1893.

George Forbes.—The Electrical Transmission of Power from Niagara Falls. (Jour. Inst. Elec. Engrs., Vol. 22, p. 484.)

1898.

A. F. McKissick.—Some Tests with an Induction Generator. (Trans. Am. Inst. Elec. Engrs., Vol. 15, p. 409.)

1899.

M. R. Gardner and R. P. Howgrave-Graham.—The Synchronizing of Alternators. (Jour. Inst. Elec. Engrs., Vol. 28, p. 658.)

1900.

B. A. Behrend.—On the Mechanical Forces in Dynamos Caused by Magnetic Attraction. (Trans. Am. Inst. Elec. Engrs., Vol. 17, p. 617.)

1901.

W. L. R. Emmet.—Parallel Operation of Engine-Driven Alternators. (Trans. Am. Inst. Elec. Engrs., Vol. 18, p. 745.)

Ernst J. Berg.—Parallel Running of Alternators. (Trans. Am. Inst. Elec. Engrs., Vol. 18, p. 753.)

P. O. Keilholtz.—Angular Variation in Steam Engines. (Trans. Am. Inst. Elec. Engrs., Vol. 18, p. 703.)

Chas. P. Steinmetz.—Speed Regulation of Prime Movers and Parallel Operation of Alternators. (Trans. Am. Inst. Elec. Engrs. Vol. 18, p. 741.)

Walter I. Slichter.—Angular Velocity in Steam Engines in Relation to Paralleling of Alternators. (Trans. Am. Inst. Elec. Engrs., Vol. 18, p. 759.)

1902.

C. O. Mailloux.—An Experiment with Single-Phase Alternators on Polyphase Circuits. (Trans. Am. Inst. Elec. Engrs., Vol. 19, p. 851.)

Louis A. Herdt.—The Determination of Alternator Characteristics. (Trans. Am. Inst. Elec. Engrs., Vol. 19, p. 1093.)

C. E. Skinner.—Energy Loss in Commercial Insulating Materials when Subjected to High-Potential Stress. (Trans. Am. Inst. Elec. Engrs., Vol. 19, p. 1047.)

1903.

C. A. Adams.—A Study of the Heyland Machine as Motor and Generator. (Trans. Am. Inst. Elec. Engrs., Vol. 21, p. 519.)

W. L. Waters.—Commercial Alternator Design. (Trans. Am. Inst. Elec. Engrs., Vol. 22, p. 39.)

A. S. Garfield.—The Compounding of Self-Excited Alternating-Current Generators for Variation in Load and Power Factor. (Trans. Am. Inst. Elec. Engrs., Vol. 21, p. 569.)

B. A. Behrend.—The Experimental Basis for the Theory of the Regulation of Alternators. (Trans. Am. Inst. Elec. Engrs., Vol. 21, p. 497.)

1904.

A. F. T. Atchison.—Some Properties of Alternators Under Various Conditions of Load. (Jour. Inst. Elec. Engrs., Vol. 33, p. 1062.)

H. W. Taylor.—Armature Reaction in Alternators. (Jour. Inst. Elec. Engrs., Vol. 33, p. 1144.)

Miles Walker.—Compensated Alternate-Current Generators. (Jour. Inst. Elec. Engrs., Vol. 34, p. 402.)

J. B. Henderson and J. S. Nicholson.—Armature Reaction in Alternators. (Jour. Inst. Elec. Engrs., Vol. 34, p. 465.)

David B. Rushmore.—The Mechanical Construction of Revolving Field Alternators. (Trans. Am. Inst. Elec. Engrs., Vol 23, p. 253.)

B. G. Lamme.—Data and Tests on a 10 000 Cycle-per-Second Alternator. (Trans. Am. Inst. Elec. Engrs., Vol. 23, p. 417.)

H. H. Barnes, Jr.—Notes on Fly-Wheels. (Trans. Am. Inst. Elec. Engrs., Vol. 23, p. 353.)

H. M. Hobart and Franklin Punga.—A Contribution to the Theory of the Regulation of Alternators. (Trans. Am. Inst Elec. Engrs., Vol. 23, p. 291.)

1905.

William Stanley and G. Faccioli.—Alternate-Current Machinery, with Especial Reference to Induction Alternators. (Trans. Am. Inst. Elec. Engrs., Vol. 24, p. 851.)

A. B. Field.—Eddy Currents in Large, Slot-wound Conductors. (Trans. Am. Inst. Elec. Engrs., Vol. 24, p. 761.)

W. J. A. London.—Mechanical Construction of Steam-Turbines and Turbo-Generators. (Jour. Inst. Elec. Engrs., Vol. 35, p. 163.)

1906.

J. Epstein.—Testing Electrical Machinery and Materials. (Jour. Inst. Elec. Engrs., Vol. 38, p. 28.)

A. G. Ellis.—Steam Turbine Generators. (Jour. Inst. Elec. Engrs., Vol. 37, p. 305.)

Sebastian Senstius.—Heat Tests on Alternators. (Trans. Am. Inst. Elec. Engrs., Vol. 25, p. 311.)

Morgan Brooks and M. K. Akers.—The Self-Synchronizing of Alternators. (Trans. Am. Inst. Elec. Engrs., Vol. 25, p. 453.)

E. F. Alexanderson.—A Self-Exciting Alternator. (Trans. Am. Inst. Elec. Engrs., Vol. 25, p. 61.)

1907.

B. A. Behrend.—Introduction to Discussion on the Practicability of Large Generators Wound for 22 000 Volts. (Trans. Am. Inst. Elec. Engrs., Vol. 26, p. 351.)

Robert Pohl.—Development of Turbo-Generators. (Jour. Inst. Elec. Engrs., Vol. 40, p. 239.)

G. W. Worrall.—Magnetic Oscillations in Alternators. (Jour. Inst. Elec. Engrs., Vol. 39, p. 206.) [This paper is supplemented by another paper contributed by Mr. Worrall in 1908.]

1908.

M. Kloss.—Selection of Turbo-Alternators. (Jour. Inst. Elec. Engrs., Vol. 42, p. 156.)

S. P. Smith.—Testing of Alternators. (Jour. Inst. Elec. Engrs., Vol. 42, p. 190.)

G. Stoney and A. H. Law.—High-Speed Electrical Machinery. (Jour. Inst. Elec. Engrs., Vol. 41, p. 286.)

R. K. Morcom and D. K. Morris.—Testing Electrical Generators. (Jour. Inst. Elec. Engrs., Vol. 41, p. 137.)

G. W. Worrall.—Magnetic Oscillations in Alternators. (Jour. Inst. Elec. Engrs., Vol. 40, p. 413.) [This paper is a continuation of Mr. Worrall's 1907 paper.]

Jens Bache-Wiig.—Application of Fractional Pitch Windings to Alternating-Current Generators. (Trans. Am. Inst. Elec. Engrs., Vol. 27, p. 1077.)

Carl J. Fechheimer.—The Relative Proportions of Copper and Iron in Alternators. (Trans. Am. Inst. Elec. Engrs., Vol. 27, p. 1429.)

1909.

S. P. Smith.—The Testing of Alternators. (Jour. Inst. Elec. Engrs., Vol. 42, p. 190.)

J. D. Coales.—Testing Alternators. (Jour. Inst. Elec. Engrs., Vol. 42, p. 412.)

E. Rosenberg.—Parallel Operation of Alternators. (Jour. Inst. Elec. Engrs., Vol. 42, p. 524.)

E. F. W. Alexanderson.—Alternator for One Hundred Thousand Cycles. (Trans. Am. Inst. Elec. Engrs., Vol. 28, p. 399.)

Carl J. Fechheimer.—Comparative Costs of 25-Cycle and 60-Cycle Alternators. (Trans. Am. Inst. Elec. Engrs., Vol. 28, p. 975.)

C. A. Adams.—Electromotive Force Wave-Shape in Alternators. (Trans. Am. Inst. Elec. Engrs., Vol. 28, p. 1053.)

1910.

Miles Walker.—Short-Circuiting of Large Electric Generators. (Jour. Inst. Elec. Engrs., Vol. 45, p. 295.)

Miles Walker.—Design of Turbo Field Magnets for Alternate-Current Generators. (Jour. Inst. Elec. Engrs., Vol. 45, p. 319.)

Geo. I. Rhodes.—Parallel Operation of Three-Phase Generators with their Neutrals Interconnected. (Trans. Am. Inst. Elec. Engrs., Vol. 29, p. 765.)

H. G. Stott and R. J. S. Pigott.—Tests of a 15 000-kw. Steam-Engine-Turbine Unit. (Trans. Am. Inst. Elec. Engrs., Vol. 29, p. 183.)

E. D. Dickinson and L. T. Robinson.—Testing Steam Turbines and Steam Turbo-Generators. (Trans. Am. Inst. Elec. Engrs., Vol. 29, p. 1679.)

1911.

J. R. Barr.—Parallel Working of Alternators. (Jour. Inst. Elec. Engrs., Vol. 47, p. 276.)

A. P. M. Fleming and R. Johnson.—Chemical Action in the Windings of High-Voltage Machines. (Jour. Inst. Elec. Engrs., Vol. 47, p. 530.)

S. P. Smith. — Non-Salient-Pole Turbo-Alternators. (Jour. Inst. Elec. Engrs., Vol. 47, p. 562.)

W. W. Firth.—Measurement of Relative Angular Displacement in Synchronous Machines. (Jour. Inst. Elec. Engrs., Vol. 46, p. 728.)

R. F. Schuchardt and E. O. Schweitzer.—The Use of Power-Limiting Reactances with Large Turbo-Alternators. (Trans. Am. Inst. Elec. Engrs., Vol. 30.)

1912.

H. D. Symons and Miles Walker.—The Heat Paths in Electrical Machinery. (Jour. Inst. Elec. Engrs., Vol. 48, p. 674.)

W. A. Durgin and R. H. Whitehead.—The Transient Reactions of Alternators. (Trans. Am. Inst. Elec. Engrs., Vol. 31.)

A. B. Field.—Operating Characteristics of Large Turbo-Generators. (Trans. Am. Inst. Elec. Engrs., Vol. 31.)

H. M. Hobart and E. Knowlton.—The Squirrel-Cage Induction Generator. (Trans. Am. Inst. Elec. Engrs., Vol. 31.)

E. M. Olin.—Determination of Power Efficiency of Rotating Electric Machines. (Trans. Am. Inst. Elec. Engrs., Vol. 31.)

D. W. Mead.—The Runaway Speed of Water-Wheels and its Effect on Connected Rotary Machinery. (Trans. Am. Inst. Elec. Engrs., Vol. 31.)

D. B. Rushmore.—Excitation of Alternating-Current Generators. (Trans. Am. Inst. Elec. Engrs., Vol. 31.)

APPENDIX II

A BIBLIOGRAPHY OF I. E. E. AND A. I. E. E. PAPERS ON THE SUBJECT OF POLYPHASE MOTORS

1888.

Nikola Tesla.—A New System of Alternate-Current Motors and Transformers. (Trans. Am. Inst. Elec. Engrs., Vol. 5, p. 308.)

1893.

Albion T. Snell.—The Distribution of Power by Alternate-Current Motors. (Jour. Inst. Elec. Engrs., Vol. 22, p. 280.)

1894.

Louis Bell.—Practical Properties of Polyphase Apparatus. (Trans. Am. Inst. Elec. Engrs., Vol. 11, p. 3.)

Louis Bell.—Some Facts about Polyphase Motors. (Trans. Am. Inst. Elec. Engrs., Vol. 11, p. 559.)

Louis Duncan, J. H. Brown, W. P. Anderson, and S. Q. Hayes.—Experiments on Two-Phase Motors. (Trans. Am. Inst. Elec. Engrs., Vol. 11, p. 617.)

Samuel Reber.—Theory of Two- and Three-Phase Motors. (Trans. Am. Inst. Elec. Engrs., Vol 11, p. 731.)

Chas. P. Steinmetz.—Theory of the Synchronous Motor. (Trans. Am. Inst. Elec. Engrs., Vol. 11, p. 763.)

Ludwig Gutmann.—On the Production of Rotary Magnetic Fields by a Single Alternating Current. (Trans. Am. Inst. Elec. Engrs., Vol. 11, p. 832.)

1897.

Chas. P. Steinmetz.—The Alternating-Current Induction Motor. (Trans. Am. Inst. Elec. Engrs., Vol. 14, p. 185.)

1899.

ERNEST WILSON.—The Induction Motor. (Jour. Inst. Elec. Engrs., Vol. 28, p. 321.)

1900.

A. C. EBORALL.—Alternating Current Induction Motors. (Jour. Inst. Elec. Engrs., Vol. 29, p. 799.)

1901.

CHAS. F. SCOTT.—The Induction Motor and the Rotary Converter and Their Relation to the Transmission System. (Trans. Am. Inst. Elec. Engrs., Vol. 18, p. 371.)

1902.

ERNST DANIELSON.—A Novel Combination of Polyphase Motors for Traction Purposes. (Trans. Am. Inst. Elec. Engrs., Vol. 19, p. 527.)

CHAS. P. STEINMETZ.—Notes on the Theory of the Synchronous Motor. (Trans. Am. Inst. Elec. Engrs., Vol. 19, p. 781.)

1903.

C. A. ADAMS.—A Study of the Heyland Machine as Motor and Generator. (Trans. Am. Inst. Elec. Engrs., Vol. 21, p. 519.)

H. BEHN-ESCHENBURG.—Magnetic Dispersion in Induction Motors. (Jour. Inst. Elec. Engrs., Vol. 33, p. 239.)

1904.

B. G. LAMME.—Synchronous Motors for Regulation of Power Factor and Line Pressure. (Trans. Am. Inst. Elec. Engrs., Vol. 23, p. 481.)

H. M. HOBART.—The Rated Speed of Electric Motors as Affecting the Type to be Employed. (Jour. Inst. Elec. Engrs., Vol. 33, p. 472.)

1905.

R. GOLDSCHMIDT.—Temperature Curves and the Rating of Electrical Machinery. (Jour. Inst. Elec. Engrs., Vol. 34, p. 660.)

D. K. MORRIS and G. A. LISTER.—Eddy-Current Brake for Testing Motors. (Jour. Inst. Elec. Engrs., Vol. 35, p. 445.)

P. D. IONIDES.—Alternating-Current Motors in Industrial Service. (Jour. Inst. Elec. Engrs., Vol. 35, p. 475.)

C. A. Adams.—The Design of Induction Motors. (Trans. Am. Inst. Elec. Engrs., Vol. 24, p. 649.)

Chas. A. Perkins.—Notes on a Simple Device for Finding the Slip of an Induction Motor. (Trans. Am. Inst. Elec. Engrs., Vol. 24, p. 879.)

A. S. Langsdorf.—Air-Gap Flux in Induction Motors. (Trans. Am. Inst. Elec. Engrs., Vol. 24, p. 919.)

1906.

J. B. Taylor.—Some Features Affecting the Parallel Operation of Synchronous Motor-Generator Sets. (Trans. Am. Inst. Elec. Engrs., Vol. 25, p. 113.)

Bradley McCormick.—Comparison of Two- and Three-Phase Motors. (Trans. Am. Inst. Elec. Engrs., Vol. 25, p. 295.)

A. Baker and J. T. Irwin.—Magnetic Leakage in Induction Motors. (Jour. Inst. Elec. Engrs., Vol. 38, p. 190.)

1907.

L. J. Hunt.—A New Type of Induction Motor. (Jour. Inst. Elec. Engrs., Vol. 39, p. 648.)

R. Rankin.—Induction Motors. (Jour. Inst. Elec. Engrs., Vol. 39, p. 714.)

C. A. Adams, W. K. Cabot, and C. A. Irving, Jr.—Fractional-Pitch Windings for Induction Motors. (Trans. Am. Inst. Elec. Engrs., Vol. 26, p. 1485.)

R. E. Hellmund.—Zigzag Leakage of Induction Motors. (Trans. Am. Inst. Elec. Engrs., Vol. 26, p. 1505.)

1908.

R. Goldschmidt.—Standard Performances of Electrical Machinery. (Jour. Inst. Elec. Engrs., Vol. 40, p. 455.)

G. Stevenson.—Polyphase Induction Motors. (Jour. Inst. Elec. Engrs., Vol. 41, p. 676.)

H. C. Specht.—Induction Motors for Multi-Speed Service with Particular Reference to Cascade Operation. (Trans. Am. Inst. Elec. Engrs., Vol. 27, p. 1177.)

1909.

J. C. MacFarlane and H. Burge.—Output and Economy Limits of Dynamo-Electric Machinery. (Jour. Inst. Elec. Engrs., Vol. 42, p. 232.)

S. B. Charters, Jr., and W. A. Hillebrandt.—Reduction in

Capacity of Polyphase Motors Due to Unbalancing in Voltage. (Trans. Am. Inst. Elec. Engrs., Vol. 28, p. 559.)

H. G. Reist and H. Maxwell.—Multi-Speed Induction Motors. (Trans. Am. Inst. Elec. Engrs., Vol. 28, p. 601.)

A. Miller Gray.—Heating of Induction Motors. (Trans. Am. Inst. Elec. Engrs., Vol. 28, p. 527.)

1910.

R. E. Hellmund.—Graphical Treatment of the Zigzag and Slot Leakage in Induction Motors. (Jour. Inst. Elec. Engrs., Vol. 45, p. 239.)

C. F. Smith.—Irregularities in the Rotating Field of the Polyphase Induction Motor. (Jour. Inst Elec. Engrs., Vol. 46, p. 132.)

Walter B. Nye.—The Requirements for an Induction Motor from the User's Point of View. (Trans. Am. Inst. Elec. Engrs., Vol. 29, p. 147.)

1911.

T. F. Wall.—The Development of the Circle Diagram for the Three-Phase Induction Machine. (Jour. Inst. Elec. Engrs., Vol. 48, p. 499.)

N. Pensabene-Perez.—An Automatic Starting Device for Asynchronous Motors. (Jour. Inst. Elec. Engrs., Vol. 48, p. 484.)

C. F. Smith and E. M. Johnson.—The Losses in Induction Motors Arising from Eccentricity of the Rotor. (Jour. Inst. Elec. Engrs., Vol. 48, p. 546.)

H. J. S. Heather.—Driving of Winding Engines by Induction Motors. (Jour. Inst. Elec. Engrs., Vol. 47, p. 609.)

Theodore Hoock.—Choice of Rotor Diameter and Performance of Polyphase Induction Motors. (Trans. Am. Inst. Elec. Engrs., Vol. 30.)

Gus A. Maier.—Methods of Varying the Speed of Alternating Current Motors. (Trans. Am. Inst. Elec. Engrs., Vol. 30.)

1912.

J. K. Catterson-Smith.—Induction Motor Design. (Jour. Inst. Elec. Engrs., Vol. 49, p. 635.)

Carl J. Fechheimer.—Self-Starting Synchronous Motors. (Trans. Am. Inst. Elec. Engrs., Vol. 31.)

H. C. Specht.—Electric Braking of Induction Motors. (Trans. Am. Inst. Elec. Engrs., Vol. 31.)

P. M. Lincoln.—Motor Starting Currents as Affecting Large Transmission Systems. (Trans. Am. Inst. Elec. Engrs., Vol. 31.)

APPENDIX III

TABLE OF SINES, COSINES, AND TANGENTS FOR USE IN THE CALCULATIONS IN CHAPTER II.

Angle.	Sin.	Cos.	Tan.	Angle.	Sin.	Cos.	Tan.
0°	0	1.000	0				
1°	0.01745	0.999	0.01745	46°	0.719	0.695	1.035
2°	0.03490	0.999	0.03490	47°	0.731	0.682	1.072
3°	0.0523	0.998	0.0524	48°	0.743	0.669	1.111
4°	0.0697	0.998	0.0619	49°	0.755	0.656	1.150
5°	0.0871	0.996	0.0874	50°	0.766	0.643	1.192
6°	0.1045	0.994	0.1051	51°	0.777	0.629	1.235
7°	0.1218	0.992	0.1227	52°	0.788	0.616	1.280
8°	0.1391	0.990	0.1405	53°	0.799	0.602	1.327
9°	0.1564	0.988	0.1583	54°	0.809	0.588	1.376
10°	0.1736	0.985	0.1763	55°	0.819	0.574	1.428
11°	0.1908	0.982	0.1943	56°	0.829	0.559	1.482
12°	0.2079	0.978	0.2125	57°	0.839	0.545	1.540
13°	0.2249	0.974	0.2308	58°	0.848	0.530	1.600
14°	0.2419	0.970	0.2493	59°	0.857	0.515	1.664
15°	0.2588	0.966	0.2679	60°	0.866	0.500	1.732
16°	0.2756	0.961	0.2867	61°	0.875	0.4848	1.804
17°	0.2923	0.956	0.3057	62°	0.883	0.4694	1.880
18°	0.3090	0.951	0.3249	63°	0.891	0.4539	1.963
19°	0.3255	0.945	0.3443	64°	0.899	0.4383	2.050
20°	0.3420	0.940	0.3639	65°	0.906	0.4226	2.144
21°	0.3583	0.934	0.3838	66°	0.914	0.4067	2.246
22°	0.3746	0.927	0.4040	67°	0.920	0.3907	2.356
23°	0.3907	0.920	0.4244	68°	0.927	0.3746	2.475
24°	0.4067	0.914	0.4452	69°	0.934	0.3583	2.605
25°	0.4226	0.906	0.4663	70°	0.940	0.3420	2.747
26°	0.4383	0.899	0.4877	71°	0.945	0.3255	2.904
27°	0.4539	0.891	0.509	72°	0.951	0.3090	3.077
28°	0.4694	0.883	0.532	73°	0.956	0.2923	3.271
29°	0.4848	0.875	0.554	74°	0.961	0.2756	3.487
30°	0.500	0.866	0.577	75°	0.966	0.2588	3.732
31°	0.515	0.857	0.601	76°	0.970	0.2419	4.011
32°	0.530	0.848	0.625	77°	0.974	0.2249	4.331
33°	0.545	0.839	0.649	78°	0.978	0.2079	4.705
34°	0.559	0.829	0.674	79°	0.982	0.1908	5.14
35°	0.574	0.819	0.700	80°	0.985	0.1736	5.67
36°	0.588	0.809	0.726	81°	0.988	0.1564	6.31
37°	0.602	0.799	0.754	82°	0.990	0.1391	7.11
38°	0.616	0.788	0.781	83°	0.992	0.1218	8.14
39°	0.629	0.777	0.810	84°	0.994	0.1045	9.51
40°	0.643	0.766	0.839	85°	0.996	0.0871	11.4
41°	0.656	0.755	0.869	86°	0.998	0.0697	14.3
42°	0.669	0.743	0.900	87°	0.998	0.0523	19.1
43°	0.682	0.731	0.932	88°	0.999	0.03490	28.6
44°	0.695	0.719	0.966	89°	0.999	0.01745	57.3
45°	0.707	0.707	1.000	90°	1.000	0	Infinite

APPENDIX IV

Metric Wire Table.

Diameter.				Cross-section.	Resistance in Ohms per Kilometer.						Meters per Ohm at 20° C.	Kilograms per Ohm at 20° C.	Ohms per Kilogram at 20° C.	Meters per Kilogram.	Kilograms per Kilometer. (Bare.)
Bare. mm.	S.C.C. mm.	D.C.C. mm.	T.C.C. mm.	sq.cm.	0° C.	20° C.	40° C.	60° C.	80° C.	100° C.					
12	.....	.....	12.50	1.135	0.1415	0.153	0.165	0.177	0.19	0.202	6550	6600	0.0001510	0.995	1010
11	.....	.....	11.40	0.950	0.1680	0.1820	0.1960	0.2100	0.226	0.2400	5500	4620	0.0002160	1.19	840
10	.....	.....	10.45	0.787	0.2035	0.22	0.237	0.254	0.273	0.29	4550	3180	0.000313	1.425	701
9	.....	.....	9.45	0.638	0.252	0.271	0.292	0.314	0.337	0.358	3690	2090	0.000477	1.76	567
8	.....	.....	8.25	0.503	0.319	0.344	0.372	0.399	0.429	0.455	2910	1300	0.000768	2.23	448
7	.....	7.35	7.45	0.385	0.4150	0.4450	0.480	0.5150	0.5550	0.5900	2245	770	0.001298	2.92	342
6	.....	6.35	6.45	0.283	0.564	0.61	0.657	0.705	0.757	0.803	1640	414	0.00241	3.97	252
5	.....	5.31	5.35	0.197	0.810	0.875	0.940	1.010	1.090	1.150	1140	200	0.00501	5.69	175.5
4	.....	4.32	4.40	0.126	1.27	1.37	1.48	1.59	1.71	1.81	730	81.9	0.01225	8.9	112
3	.....	3.28	3.40	0.0710	2.26	2.44	2.63	2.81	3.03	3.21	410	25.9	0.0387	15.8	63.2
2	2.15	2.26	2.35	0.0316	5.09	5.48	5.88	6.32	6.80	7.22	182.5	5.11	0.1970	35.5	28.2
1	1.122	1.19	1.30	0.00794	20.4	22.0	23.7	25.4	27.3	29 0	45.5	0.320	3.13	142	7.0
0.9	0.990	1.09	1.18	0.00635	25.0	27.0	29.3	31.4	33.8	35.7	37.0	0.210	4.79	176	5.6
0.8	0.890	0.990	1.09	0.00502	31.6	34.3	37.2	39.8	42.8	45.0	29.1	0.130	7.74	223	4.48
0.7	0.801	0.890	1.00	0.00387	41.2	44.5	48.2	51.7	55.5	58.6	22.5	0.077	13.05	290	3.42
0.6	0.701	0.812	0.900	0.00283	56.2	61.0	66.0	70.8	75.8	80.0	16.4	0.041	24.40	398	2.5
0.5	0.602	0.710	0.790	0.00196	81.3	88.0	95	102	109.0	116.0	11.4	0.0198	50.70	580	1.74
0.4	0.504	0.610	.....	0.00125	128	137.0	149	160	172.0	182.0	7.3	0.00915	123.9	894	1.12
0.3	0.412	0.508	.....	0.000705	227	245	264	285	305	320	4.1	0.00258	388	1580	0.625
0.2	0.290	.....	.....	0.000314	510	548	594	640	680	720	1.82	0.000505	1980	3600	0.277
0.1	.....	.....	.....	0.0000794	204	2200	2370	2540	2730	2900	0.455	0.000032	31300	14200	0.07

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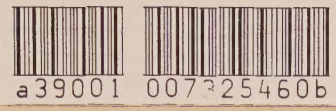
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